Inequality Proof
Let the function $f : \mathbb{Z}^+ \to \mathbb{R}$ be defined by

$$f(1) = 2$$
$$f(n) = \frac{1}{2}f(n-1) + \frac{5}{2f(n-1)}$$

For any integer $k \geq 2$, prove by direct proof that if $2 \leq f(k-1) \leq \frac{5}{2}$, then $2 \leq f(k) \leq \frac{5}{2}$.

Thinking about Big-O

Prof. Flitwick claims that for any functions f and g from the reals > 1 to the reals > 1. If $f(x) \ll g(x)$ then $\log(f(x)) \ll \log(g(x))$. Is this true? Briefly justify your answer.

More thinking about Big-O

Suppose that f, g, and h are functions from the reals to the reals, such that f is $O(g)$ and g is $O(h)$. Must f be $O(h)$? Briefly justify your answer.