

Nested Quantifiers

Explain why each of the following propositions is true or false:

1. $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, xy = x$
2. $\exists y \in \mathbb{Q}, \forall x \in \mathbb{R}, |x - y| \leq 0.01$

Do at least one of the following.

Function Proof

Suppose that $f : \mathbb{N} \rightarrow \mathbb{N}$ is an onto function. Let's define $g : \mathbb{N}^2 \rightarrow \mathbb{N}$ such that $g(x, y) = f(x)f(y)$.

Prove that g is onto.

or

Suppose that $f : \mathbb{Z} \rightarrow \mathbb{N}$ such that

$$f(x) = \begin{cases} f(n) = 2n, & \text{if } n \geq 0 \\ f(n) = -(2n + 1), & \text{if } n < 0 \end{cases}$$

Prove that f is one-to-one.

Hint: use two cases, based on whether the output values are even or odd.