

Thinking about gcd

Is the following claim true or false. Give a counter-example (if false) or an informal explanation (if true).

For any positive integers p, q , and r , if $\gcd(p, q) = 1$ and $\gcd(q, r) = 1$, then $\gcd(p, r) = 1$.

Proof

Prove the following:

For all integers x, y, p, q , and m , with $m > 0$, if $x \equiv p \pmod{m}$ and $y \equiv q \pmod{m}$, then $(x^2 + y^2) \equiv (p^2 + q^2) \pmod{m}$.

Use your best mathematical style and the following definition of congruence mod k : $x \equiv y \pmod{k}$ if and only if $x = y + nk$ for some integer n .