Grammar Trees

Define a grammar G_1 by $S \rightarrow aSbS \mid SaS \mid ab \mid a$. where S is the only start symbol and the terminal symbols are a and b. Prove that a tree generated by G_1 has at least as many nodes labeled a as nodes labeled b.

Solution:

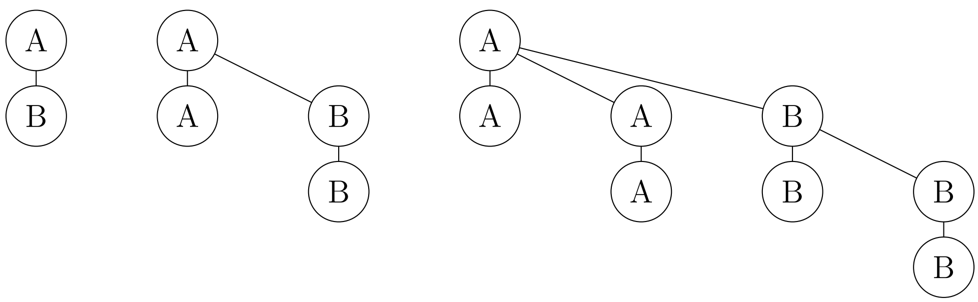
M-nomial Trees

A **m-nomial** of order m is defined recursively as follows:

(1) A single root node is a m-nomial tree of order 0.

(2) A m-nomial tree of order m consists of two m-nomial trees of order $m-1$, with the root of the second connected as the rightmost child of the root of the first.

The following picture shows the m-nomial trees of order 1, 2, and 3. The labels on the nodes show how the larger tree is divided into two lower-order subtrees.



Use induction on the order of the tree to prove that a m-nomial tree of order m has 2^m nodes.

Solution:

Parity Trees

A parity tree is a full binary tree with each node colored orange or blue such that:

1. If v is a leaf node, then v is colored orange.
2. If v has two children of the same color, then v is colored blue.
3. If v has two children of different colors, then v is colored orange.

Prove by induction that every parity tree has the parity property that if the root is colored orange, then it has an odd number of leaves; and if the root is colored blue, then it has an even number of leaves.

Solution: