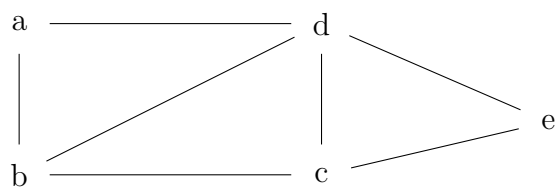


**Paths**

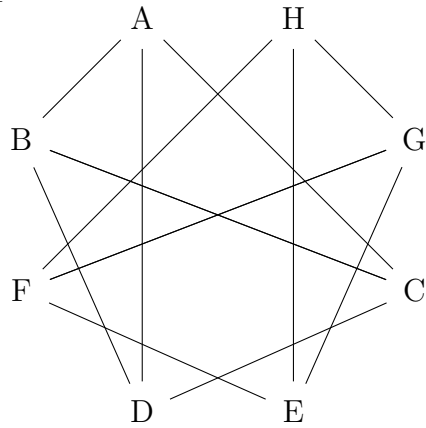
List all the paths from  $b$  to  $e$  in graph  $G$  below.

 $G$ 

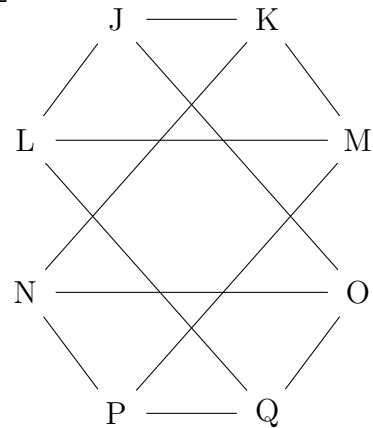
**Solution:**

**Component** Is each of these graphs connected? If not, list the nodes in each connected component.

$G_1$ :

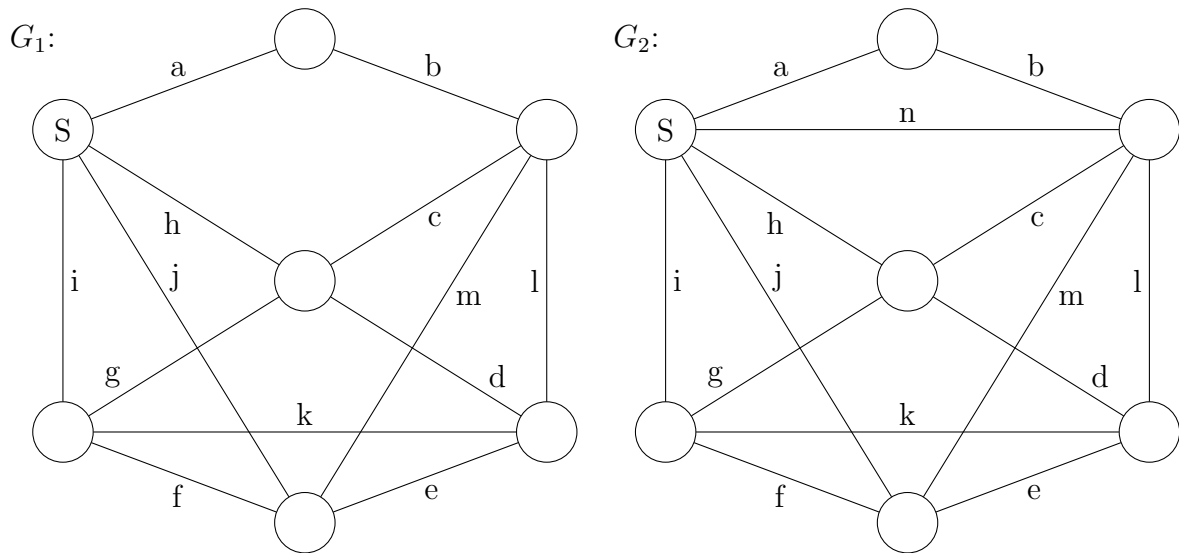


$G_2$ :



**Solution:**

**Euler Circuits** Find an Euler circuit in each graph beginning at  $S$ , or explain why this isn't possible.



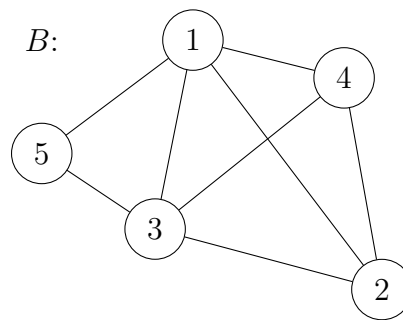
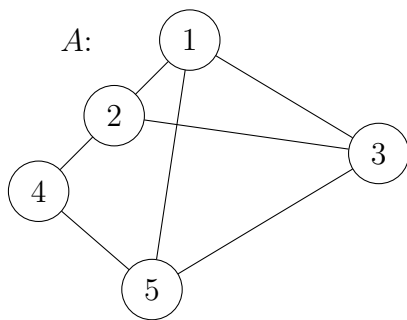
**Solution:**

**Chromatic Number**

Recall that the justification that a particular chromatic number is valid requires bounding the number from above *and* below. Therefore you must give an *explicit* coloring to produce an upper bound *and* produce a valid argument that no smaller number of colors will work to produce a lower bound.

The argument justifying the lower bound often involves finding a copy of  $K_n$  (where  $n$  is the chromatic number you are attempting to validate) as a subgraph. Sometimes, however, you have to work through the space of possible  $n - 1$  colorings by hand and show that none of them work.

Find and justify the chromatic numbers for each of the following graphs.



**Solution:**