

23<sup>rd</sup> of September, 2025

1. Consider the function  $f : \mathbb{R} \times \mathbb{N} \rightarrow \mathbb{Z}$  given by  $f(x, y) := \left\lfloor \frac{x}{y+1} \right\rfloor$ .
  - (a) Is  $f$  a one-to-one function?

- (b) Is  $f$  an onto function?

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2. Let  $f : \mathbb{N} \rightarrow \mathbb{N}$  be a one-to-one function.

Consider the function  $g : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{Z} \times \mathbb{Z}$  given by  $g(x, y) := (f(x) - y, f(x) + y)$ .

(a) Is  $g$  a one-to-one function?

(b) Is  $g$  an onto function?

3. Consider the function  $h : \mathbb{Z} \rightarrow \mathbb{N}$  given by  $h(x) = \begin{cases} 2x & \text{if } x \geq 0 \\ -2x - 1 & \text{if } x < 0 \end{cases}$ .

(a) Is  $h$  a one-to-one function?

(b) Is  $h$  an onto function?