Power Sets
Define the following sets:

$$\begin{aligned} A &= \{68, 28\} \\ B &= \{\text{rain}, \text{snow}, \text{sun}\} \\ C &= \{\text{water}, \text{ice}\} \\ D &= \{\{\text{water}\}, \{\text{milk}\}\} \\ E &= \{(\text{water}, \text{ice})\} \\ F &= \{\text{ink}\} \end{aligned}$$

List the elements of each of the following sets or calculate the cardinality (as indicated).

(a) $\mathbb{P}(B)$

(b) $\mathbb{P}(E)$

(c) $\mathbb{P}(C) - D$

(d) $\mathbb{P}(C) \cap \mathbb{P}(E)$

(e) $|\mathbb{P}(A \cup B) \cup \mathbb{P}(D \cup E)|$

Solution:

Partitions

Recall that a partition $\mathcal{P}$ of a (finite) set S is a collection of subsets (denoted $S_1, \ldots, S_n$) of S that satisfies the following three properties:

(1) $\mathcal{P}$ covers all of $S : S_1 \cup S_2 \cup \ldots \cup S_n = S$

(2) $\mathcal{P}$ contains no empty sets: $S_i \neq \emptyset$ for all $i \in \{1, \ldots, n\}$

(3) $\mathcal{P}$ contains no overlapping sets: $S_i \cap S_j = \emptyset$ whenever $i \neq j$

Suppose that $S = \{a, b, c, d, e, f, g\}$. Determine whether each of the following sets is a partition of S. Explain why or why not.

(a) $\{\{c, b, f\}, \{a, g\}, \{e\}, \{d\}\}$

(b) $\{\{c, b, f\}, \{b, d, e\}, \{a, g\}, \emptyset\}$

(c) $\{\{c, b, f\}, \{a, g\}, \{e\}, \{d\}, \{\emptyset\}\}$

(d) $\{\{c, h, f\}, \{d, e\}, \{a, g, b\}\}$

(e) $\{\{a, b, c, d, e, f, g\}\}$

(f) $\{\{\{a, b, c, d\}\}, \{\{e, f, g\}\}\}$

Solution:

Counting One

You need to form a battle group of 11 made up of orcs, elves, and goblins. In how many ways can you choose the composition of your battle group?

Solution:

Counting Two

You need to form a battle group consisting of an orc, an elf, and a goblin whose total strength is 12. The strength of each creature is an integer between 1 and 10 and the strength of the group is the sum of the individual strengths. In how many ways can you construct your battle group?

Solution: