1. Let $A := \{x \in \mathbb{N} \mid \exists n \in \mathbb{N}, (n > 1) \wedge (x = 2^n)\}$ and $B := \{x \in \mathbb{Z} \mid x \equiv 0 \pmod 2\}$. Show that $A \subseteq B$.

   Proof. Let $x \in A$. This tells us $x \in \mathbb{N}$ and there exists $n \in \mathbb{N}$ such that $n > 1$ and $x = 2^n$ by definition. We can now observe the following.
   $$x = 2^n = 2 \cdot 2^{n-1}$$
   Since $n \in \mathbb{N}$ and $n > 1$, we can see $n - 1 \in \mathbb{N}$, so that $2^{n-1} \in \mathbb{Z}$. This lets us deduce $2 \mid x$ by definition. Since $x = x - 0$, this means $2 \mid x - 0$, so that $x \equiv 0 \pmod 2$ by definition. Therefore, $x \in B$ by definition. We can then conclude that $A \subseteq B$. Q.E.D.

2. Show that, for any sets A, B, and C, we have $(A - B) \times C \subseteq (A \times C) - (B \times C)$.

   Proof. Let A, B, and C be arbitrary sets and let $(x, y) \in (A - B) \times C$, so that $x \in A - B$ and $y \in C$ by definition. Since $x \in A - B$, we know $x \in A$ and $x \notin B$. Noticing that $x \in A$ and $y \in C$, we can deduce $(x, y) \in A \times C$. Now, observe the following chain of reasoning.
   $$\begin{aligned} x \notin B &\Rightarrow (x \notin B) \vee (y \notin C) \\ &\Rightarrow \neg(x \in B \wedge y \in C) \\ &\Rightarrow (x, y) \notin B \times C \end{aligned}$$
   As a result, we have $(x, y) \notin B \times C$. Combining this with our previous deduction, we can conclude that $(x, y) \in (A \times C) - (B \times C)$. Therefore, $(A - B) \times C \subseteq (A \times C) - (B \times C)$ as desired. Q.E.D.

3. Let $A := \{x \in \mathbb{Z} \mid x \equiv 0 \pmod 2\}$ and $B := \{x \in \mathbb{Z} \mid x \equiv 0 \pmod 4\}$. Disprove $A \subseteq B$.

   Proof. Consider the integer $2 \in \mathbb{Z}$ and observe that $2 - 0 = 2 \cdot 1$. Since $1 \in \mathbb{Z}$, this means $2 \mid 2 - 0$, so that $2 \equiv 0 \pmod 2$, so that $2 \in A$ by definition.

   We will now show that $\forall x \in \mathbb{Z}, 4x \neq 2$ by taking cases. Let $z \in \mathbb{Z}$.

   Case 1:
   Suppose $z < 0$. Since $z \in \mathbb{Z}$, this implies $z \leqslant -1$, so we have $4z \leqslant -1 < 2$. Thus, $4z \neq 2$.

   Case 2:
   Suppose $z = 0$. Then, $4z = 4 \cdot 0 = 0 \neq 2$.

   Case 3:
   Suppose $z > 0$. Since $z \in \mathbb{Z}$, this implies $z \geqslant 1$, so we have $4z \geqslant 4 \cdot 1 = 4 > 2$. Thus, $4z \neq 2$.

   Thus, in all cases, we obtained $4z \neq 2$. We therefore know $\forall x \in \mathbb{Z}, 4x \neq 2$, which is equivalent to $4 \nmid 2$, meaning $2 \not\equiv 0 \pmod 4$ by definition. This tells us that $2 \notin B$.

   Since $2 \in A$ and $2 \notin B$, we conclude $\exists x, x \in A \wedge x \notin B$ as desired. Q.E.D.