9^{th} of September, 2025

1. Let $A \coloneqq \left\{ x \in \mathbb{N} \,\middle|\, \exists n \in \mathbb{N}, (n > 1) \wedge (x = 2^n) \right\}$ and $B \coloneqq \left\{ x \in \mathbb{Z} \,\middle|\, x \equiv 0 \pmod 2 \right\}$. Show that $A \subseteq B$.

2. Show that, for any sets A, B, and C, we have $(A - B) \times C \subseteq (A \times C) - (B \times C)$.

3. Let $A \coloneqq \left\{x \in \mathbb{Z} \;\middle|\; x \equiv 0 \pmod 2\right\}$ and $B \coloneqq \left\{x \in \mathbb{Z} \;\middle|\; x \equiv 0 \pmod 4\right\}$. Disprove $A \subseteq B$.