

The Diagonal Robot

A robot is walking around on the 2D integer grid. It starts at $(1, 1)$, and at each step it moves to one of the closest diagonal grid points - e.g. its first step can take it to any of $(2, 2)$, $(2, 0)$, $(0, 0)$, or $(0, 2)$. Prove that the robot can never reach the point $(0, 1)$.

Hint:

- 1. First, draw a picture to make sure the problem statement makes sense, and experiment with what the robot can reach in a few steps. For example, find a sequence of steps that allows the robot to reach $(-3, 3)$.*
- 2. Based on patterns you see in step 1, guess some property which is true of all points the robot can reach - as a (wrong) example, you might guess that every (x, y) point the robot can reach will satisfy $x \leq y$.*
- 3. Prove by induction that your guess from the previous step is correct. Your induction variable should be the number of steps the robot takes.*
- 4. Conclude by showing that $(0, 1)$ does not have your proven property, so it must not be reachable.*