Lecture 7: Sets

Date: September 11, 2019.

Set: An unordered collection of objects.

$\emptyset = \{\}$ emptyset
$A = \{0, 2, 4, 6\}$
$B = \{\mathsf{B, C, D, E, F, J, K, P, Q, R, S, T, V}\}$
$C = \{\{0\}, \{2\}, \{4\}, \{6\}\}$
$D = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

$\mathbb{N} = \{0, 1, 2, \ldots\}$
$\mathbb{Z} = \{0, -1, 1, -2, 2, \ldots\}$
$\mathbb{Q}$ = Rationals
$\mathbb{R}$ = Real
$\mathbb{C}$ = complex numbers

Membership: A set is defined by its **members**. $x \in A$ means "x is a member of A".

Question 1. Which of the following are true?

1. (a) $0 \in \emptyset$, (b) $\emptyset \in \emptyset$, (c) $\mathsf{A} \in \emptyset$? All are false
2. (a) $0 \in A$, T (b) $\{0\} \in A$, (c) $\emptyset \in A$?
3. (a) $0 \in C$, (b) $\{0\} \in C$ T (c) $\{\{0\}\} \in C$?
4. (a) $\emptyset \in D$, (b) $\{\emptyset\} \in D$, (c) $\{\{\emptyset\}\} \in D$? F $\{\phi, \{\phi\}\}$

Containment: $A \subseteq B$ (A is contained in B) iff $\forall x[x \in A \text{ IMPLIES } x \in B]$. $A \subseteq B$ means A subset of B

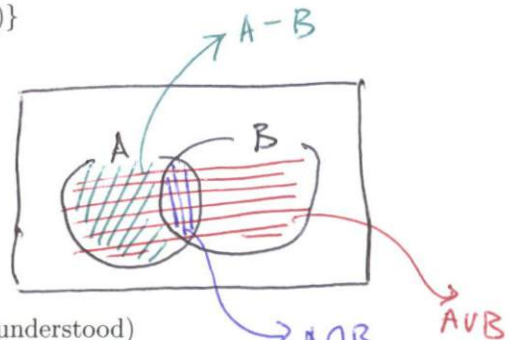
Question 2. Which of the following are true?

$\emptyset \subseteq \emptyset$ T $\emptyset \subseteq \mathbb{N}$ T → Prop: For any set X, $\phi \subseteq X$.
Picking any x. $x \notin \phi$. So Prop holds vaccuously.
$\mathbb{N} \subseteq \mathbb{N}$ T
$C \subseteq A$ F $A \subseteq C$ F Prop: For any set X, $X \subseteq X$
Pick any x. Assume $x \in X$. So prop holds

Set Builder Notation: $\{x \in A \mid P(x)\}$ defines the set of elements in A such that $P(x)$ is true.

$$E = \{n \in \mathbb{N} \mid n \text{ is even}\} = \{n \in \mathbb{N} \mid \exists k \in \mathbb{N}(n = 2k)\}$$
$$F = \{x \in \mathbb{R} \mid \exists a, b \in \mathbb{Z}(b \neq 0) \text{ AND } (x = \tfrac{a}{b})\} = \mathbb{Q}$$

Set Operations: Let X and Y be sets. OR : $\vee$ AND : $\wedge$



$X \cup Y = \{x \mid (x \in X) \text{ OR } (x \in Y)\}$

$X \cap Y = \{x \mid (x \in X) \text{ AND } (x \in Y)\}$

$X - Y = \{x \mid (x \in X) \text{ AND } (x \notin Y)\}$

$\overline{X} = U - X$, where U is the "universal set/domain of discourse" (when understood)

Question 3. What is

$A \cup C = \{0, 2, 4, 6, \{0\}, \{2\}, \{4\}, \{6\}\}$ $A \cap C = \emptyset$ $A - C = A$
$C \cap \emptyset = \emptyset$ Prop: For any set X, $\emptyset \cup X = X$
$C \cup \emptyset = C$ $\phi \cap X = \emptyset$
$\emptyset \cap D = \emptyset \neq \{\emptyset\}$

Cartesian Product: $X \times Y$ consists of all ordered pairs (x, y) where $x \in X$ and $y \in Y$, i.e., $X \times Y = \{(x, y) \mid (x \in X) \text{ AND } (y \in Y)\}$.

Example 1. $\{0, 1, 2\} \times \{a, b, c\} = \{(0,a), (0,b), (0,c), (1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$
$\{a, b, c\} \times \{0, 1, 2\} = \{(a,0), (a,1), (a,2), (b,0), (b,1), (b,2), (c,0), (c,1), (c,2)\}$
$\emptyset \times D =$
$A \times C =$

Power Set: $\mathsf{pow}(X) = \{Y \mid Y \subseteq X\}$ $P(X)$, 2^X

Question 4. $\mathsf{pow}(\{0, 1, 2\}) =$ $\{\emptyset, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\}$
$\mathsf{pow}(\emptyset)$ is (a) $\emptyset$, (b) $\{\emptyset\}$, (c) $\{\emptyset, \{\emptyset\}\}$, (d) not defined.
$\mathsf{pow}(\{\emptyset\})$ is (a) $\emptyset$, (b) $\{\emptyset\}$, (c) $\{\emptyset, \{\emptyset\}\}$, (d) not defined.

Set Equality: Two sets X and Y are equal if they have the same elements, i.e., for every x, $x \in X$ IFF $x \in Y$, i.e., $X \subseteq Y$ AND $Y \subseteq X$.

Problem 1. Prove that for any sets X, Y, Z,

$$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z).$$

Cardinality (of finite sets): $|X|$ = number of elements in X.

Example 2. $|\emptyset| =$ $|A| =$ $|D| =$ $|\{0, 1, 1, 2, 2\}| =$
$|A \times B| =$