Lecture 3: Propositional and First order Logic

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Recap: P IMPLIE Q ≡ NOT(P) OR Q

NOT(Q) IMPLIES NOT(P) ≡ NOT(NOT(Q)) OR NOT(P) ≡ Q OR NOT(P)

- A **proposition** is a statement that is either true or false. NOT(P) OR Q
- If P and Q are propositions, then so are NOT(P), P AND Q, P OR Q, P IMPLIES Q, and P IFF Q. The meaning of these combined propositions is given using a truth table. (P IMPLIES Q) AND (Q IMPLIES P)
- Two expressions are logically equivalent if they evaluate to the same truth value in **all** situations, i.e., in **every** row of the truth table they take the same value.
- The **contrapostive** of an implication P IMPLIES Q is ≡ (NOT(Q)) IMPLIES (NOT(P)). The contrapositive is **logically equivalent** to the implication.
- The **converse** of an implication P IMPLIES Q ~~is~~ ≢ Q IMPLIES P. The converse is **not logically equivalent** to the implication.

NOT(P IMPLIE Q) ≡ NOT(NOT(P) OR Q) = NOT(NOT(P)) AND NOT(Q) = P AND NOT(Q)

logically equivalent or "means the same"

Useful Logical Equivalences

NOT(NOT(P)) ≡ P	P OR Q ≡ Q OR P] COMMUTATIVITY
DE MORGAN LAWS [NOT(P OR Q) ≡ (NOT(P)) AND (NOT(Q))	P AND Q ≡ Q AND P
NOT(P AND Q) ≡ (NOT(P)) OR (NOT(Q))	P AND (Q AND R) ≡ (P AND Q) AND R] ASSOCIATIVITY
NOT(P IMPLIES Q) ≡ P AND (NOT(Q))	P OR (Q OR R) ≡ (P OR Q) OR R
P OR (Q AND R) ≡ (P OR Q) AND (P OR R)	P AND (Q OR R) ≡ (P AND Q) OR (P AND R)

Question 1.
1. What is T OR P? Is it (a) T, (b) F, or (c) P?
2. What is F OR P? Is it (a) T, (b) F, or (c) P?
3. What is T AND P? Is it (a) T, (b) F, or (c) P?
4. What is F AND P? Is it (a) T, (b) F, or (c) P?
5. Are P IMPLIES (Q IMPLIES R) and (P IMPLIES Q) IMPLIES R equivalent? NOT EQ.

P Q R	$P \to (Q \to R)$	$(P \to Q) \to R$
F F F	T	T F

Definition 1. A formula is **valid** if it is always true, no matter what truth values are assigned to the variables.

A formula is **satisfiable** if there is some truth assignment to the variables under which it evaluates to true.

Question 2. For each of the following expressions, determine if it is satisfiable or valid. T OR F ≡ T
(a) P OR Q, (b) P OR (NOT(P)), (c) P AND (NOT(P)). NEITHER SAT OR VALID
SAT SAT & VALID

Question 3. Is the following statement true? If an expression φ is satisfiable then φ is valid. F

If φ is valid then φ is satisfiable ✓

Question 4. Suppose φ is valid. Then NOT(φ) is (a) satisfiable, (b) valid, $\boxed{\text{(c) not satisfiable.}}$

φ is SAT NOT(Q) If φ is SAT then NOT(φ) is not valid

P OR NOT(P)
P OR Q

Given expression φ, is φ satisfiable

Building a truth table for φ

If φ depends n variables then truth table has 2^n

<u>QUESTION</u>: Is there a better algorithm?

$$P \stackrel{?}{=} NP$$