Name:____

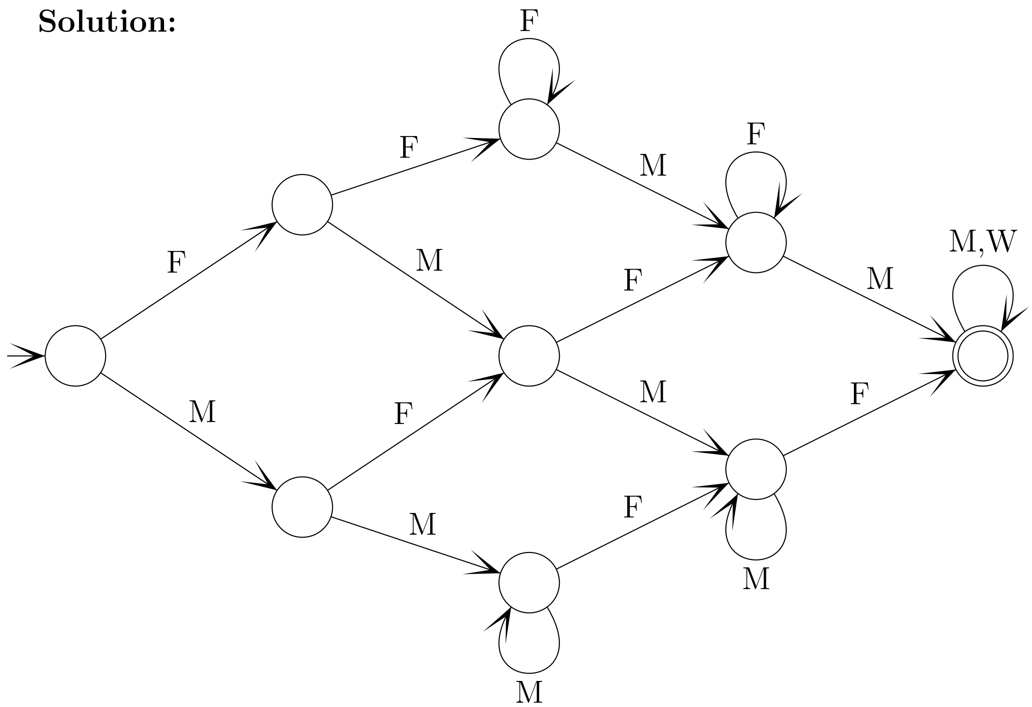
NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) When wizards enter the Magical Senate, the scanner reads M for a male wizard and F for a female wizard. The Magical Senate cannot do business unless at least two male wizards and two female wizards (W) are present. Draw a state machine that reads a sequence of M's and F's from the scanner. When it has seen two of each, it should enter an end state and stay there.

For efficiency, your state machine must be deterministic. Specifically, if you look at any state s and any action a, there is *exactly* one edge labelled a leaving state s. It should use no more than 12 states and, if you can, no more than 9.

Solution:



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(5 points) Let M be the set of all infinite-length bit vectors (i.e. strings of digits from the set $\{0, 1\}$). Let N be the set of all infinite-length strings with digits from the set $\{0, 1, 2, 3\}$, Do M and N have the same cardinality? Briefly justify your answer.

Solution: Yes, M and N have the same cardinality. Suppose we take each element of M and group its digits into pairs. Since there are four choices for each pair, we can set up a mapping between pairs of binary digits and the 4 digits used in N. Then each string in M corresponds to exactly one string in N.

(10 points) Check the (single) box that best characterizes each item.

All infinite-length strings using a finite alphabet A.	finite ☐	countably infinite ☐	uncountable ☑
If $\mathbb{P}(A)$ is uncountable, then is A infinite?	always ☑	sometimes ☐	never ☐
All walks in one fixed (finite) graph G.	finite ☐	countably infinite ☑	uncountable ☐
The set of all polynomials with real coefficients.	finite ☐	countably infinite ☐	uncountable ☑
Every function from $\{1, 2, 3\}$ to the reals has a finite formula.	true ☐	false ☑	not known ☐

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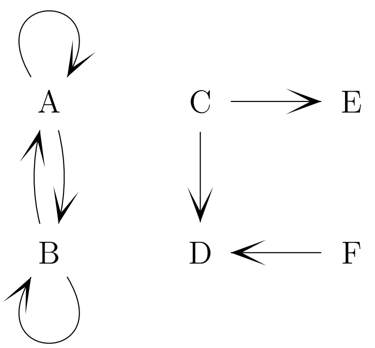
(5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: [ ] Irreflexive: [ ]

Symmetric: [ ] Antisymmetric: [ ]

Transitive: [√]

(10 points) Check the (single) box that best characterizes each item.

$\neg(p \to q) \equiv \neg p \to \neg q$ true [ ] false [√]

$\emptyset \times \emptyset =$ $\emptyset$ [√] $\{\emptyset\}$ [ ] $\{\emptyset, \emptyset\}$ [ ] $\{(\emptyset, \emptyset)\}$ [ ]

For any positive integers p, q, and k, if $p \equiv q \pmod{k}$, then $p^2 \equiv q^2 \pmod{k}$ true [√] false [ ]

The composition of two onto functions is onto. true [√] false [ ]

Chromatic number of a graph with D vertices $= D$ [ ] $= D + 1$ [ ] $\leq D + 1$ [ ] $\leq D$ [√]

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(5 points) Consider the following grammar with start symbol S and terminal symbols a and b. What strings does it generate? Be precise and briefly justify your answer.

$S \rightarrow A\ B$

$A \rightarrow a\ A \mid a$

$B \rightarrow b\ B \mid b$

Solution: The second rule generates all sequences of one or more a's. The third rule generates all sequences of one or more b's. So the start symbol S will generate one or more a's followed by one or more b's.

(10 points) Check the (single) box that best characterizes each item.

Suppose f and g produce only positive outputs and $f(n) \ll g(n)$. Will $f(n)$ be $\Theta(g(n))$?

no [√] perhaps [ ] yes [ ]

All ways to assign True/False values to n input variables

$\Theta(\log n)$	[ ]	$\Theta(n)$	[ ]	$\Theta(n \log n)$	[ ]	$\Theta(n^2)$	[ ]
$\Theta(n^3)$	[ ]	$\Theta(n^{\log_3 2})$	[ ]	$\Theta(n^{\log_2 3})$	[ ]	$\Theta(2^n)$	[√]

$T(1) = d$
$T(n) = 2T(n/2) + c$

$\Theta(n)$	[√]	$\Theta(n \log n)$	[ ]	$\Theta(n^2)$	[ ]	$\Theta(n^3)$	[ ]
$\Theta(n^{\log_3 2})$	[ ]	$\Theta(n^{\log_2 3})$	[ ]	$\Theta(2^n)$	[ ]	$\Theta(3^n)$	[ ]

A full m-ary tree with i internal nodes has $mi + 1$ nodes total.

always [√] sometimes [ ] never [ ]

$\binom{n}{1}$

-1 [ ] 0 [ ] 1 [ ] 2 [ ] n [√] undefined [ ]