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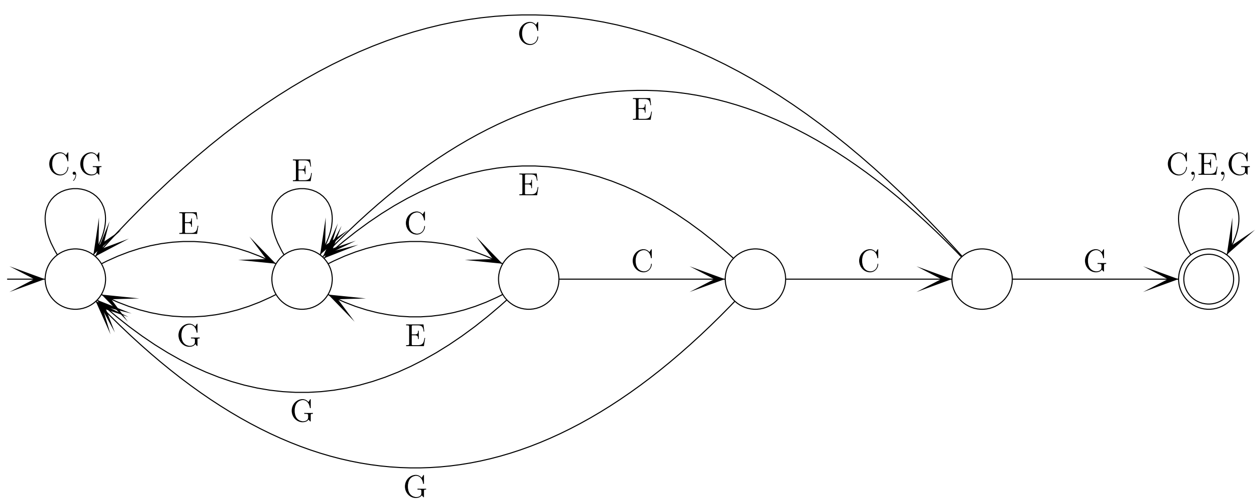
(15 points) Q is building a keychain that will explode when James Bond whistles "ECCCG". Design a finite state machine for Q that listens to an extended sequence of musical notes and enters its end state when it hears this sequence. It should remain in that end state as further notes are heard. For efficiency, the state machine must be deterministic. Specifically, if you look at any state s and any action a, there is **exactly** one edge labelled a leaving state s. Do not include the boring edges that return to the start state upon hearing a note other than C, E, or G.

Draw a deterministic state diagram that will meet his needs, using no more than 9 states and, if you can, no more than 6.

Solution:



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(5 points) Amber Industries makes a line of necklaces featuring a mama brown bear leading a line of baby bears. Customers can select any number (but only a finite number) of baby bears. Each baby bear can be chosen to have brown or cream fur. Assuming we don't worry about any practical limits on resources or necklace size, is the set of possible necklaces countable or uncountable? Briefly justify your answer.

Solution: This set is countable. Each necklace can be represented as a finite-length binary string. We know that the set of all finite-length binary strings is countable.

(10 points) Check the (single) box that best characterizes each item.

There are mathematical functions that don't have a finite formula.	true ☑	false ☐	not known ☐
The set $\mathbb{Q}^2$	finite ☐	countably infinite ☑	uncountable ☐
A subset of an uncountable set is uncountable.	true ☐	false ☑	
All partitions of the set of natural numbers less than 10,000.	finite ☑	countably infinite ☐	uncountable ☐
All line segments in the real plane.	finite ☐	countably infinite ☐	uncountable ☑

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(5 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

For any sets A, B, and C, if $A \times C \subseteq B \times C$, then $A \subseteq B$.

Solution: This is false. Suppose that $A = \{1, 2\}$, $B = \{10, 11\}$, and $C = \emptyset$. Then $A \times C = \emptyset = B \times C$, so $A \times C \subseteq B \times C$. But $A \not\subseteq B$.

(10 points) Check the (single) box that best characterizes each item.

$p \rightarrow q \equiv \neg p \rightarrow \neg q$ — true ☐ false ☑ √

If p, q, and k are primes, then $gcd(pq, qk) =$ — q ☐ pq ☐ pqk ☐ $q\,\gcd(p,k)$ ☑ √

$f : \mathbb{Z} \rightarrow \mathbb{Z}$
$f(x) = x + 3$ (x even),
$f(x) = x - 22$ (x odd) — onto ☐ not onto ☑ √ not a function ☐

Number of edges in $K_{3,4}$. — 7 ☐ 12 ☑ √ 14 ☐ 49 ☐

Suppose I want to estimate $\frac{103}{20}$. 10 is _____ — an upper bound ☑ √ an exact answer ☐ a lower bound ☐ not a bound on ☐

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(5 points) Suppose we have a function f defined by $f(1) = 5$ and $f(n) = 2f(n/2) + n^2$ (for $n \geq 2$). Express $f(n)$ in terms of $f(n/4)$. Assume n is a power of 2 and ≥ 4. Show your work and simplify your answer.

Solution:

$f(n) = 2f(n/2) + n^2 = 2(2f(n/4) + (n/2)^2) + n^2 = 4f(n/4) + \frac{n^2}{2} + n^2 = 4f(n/4) + \frac{3}{2}n^2$

(10 points) Check the (single) box that best characterizes each item.

$n!$ — $O(2^n)$ ☐ $\Theta(2^n)$ ☐ neither of these ☑

V is the vertex set of a tree with n edges. $|\mathbb{P}(V)| =$ — 2^{n-1} ☐ 2^n ☐ not determined ☐ 2^{n+1} ☑ n ☐

The running time of the Towers of Hanoi solver is recursively defined by $T(1) = d$ and $T(n) =$ — $2T(n-1) + c$ ☑ $2T(n-1) + cn$ ☐ $2T(n/2) + c$ ☐ $2T(n/2) + cn$ ☐

The number of nodes in a binary tree of height h — $\geq 2^h$ ☐ $2^{h+1} - 1$ ☐ $\leq 2^{h+1} - 1$ ☑ $\geq 2^{h+1} - 1$ ☐

$|\ \mathbb{P}(\{4,5,6,7,8\} \times \emptyset)\ |$ — $\emptyset$ ☐ $\{\emptyset\}$ ☐ 0 ☐ 1 ☑ 25 ☐ 2^5 ☐