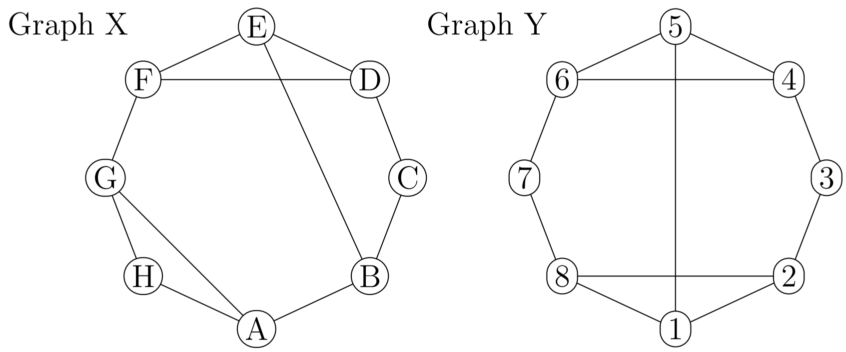
CS 173, Fall 17 Examlet 6, Part A

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

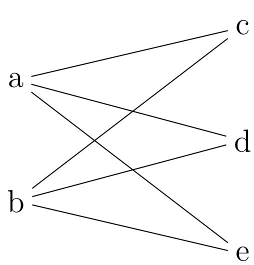
1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.



Solution: No, they are not isomorphic. In graph X, one of the degree-2 nodes is part of a 3-cycle. In graph Y, neither degree-2 node is part of a 3-cycle.

2. (5 points) Draw a picture of the graph $K_{2,3}$.

Solution:



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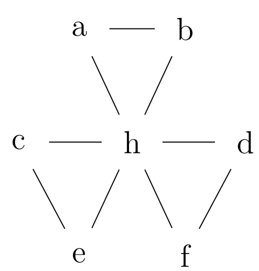
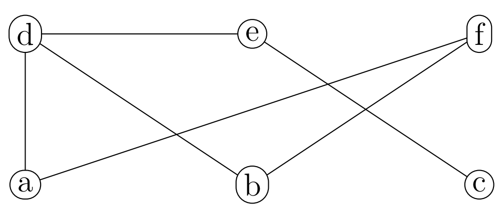
1. (10 points) How many isomorphisms are there from G (below) to itself? Justify your answer and/or show your work clearly .



Solution: Node h must map to itself. The three triangles can be permuted: 3! choices. Then each triangle can optionally be flipped over: 2 choices for each triangle. So we have a total of $3! \cdot 2^3$ isomorphisms.

2. (5 points) Is this graph bipartite? Briefly justify your answer.



Solution: Yes, this is bipartite. Suppose we put nodes d, c, and f in the first set, and nodes a, b, and e in the second set. Then all edges connect a node from the first set to a node from the second set.

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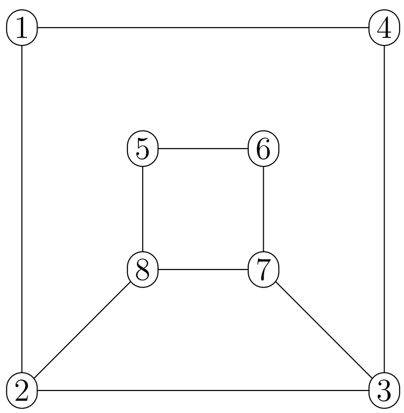
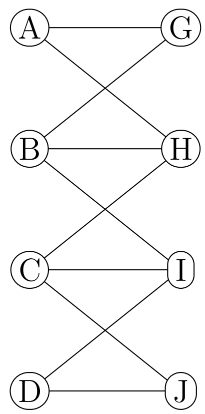
1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



Solution: Yes, X and Y are isomorphic.

For example, use the node mapping $f(1) = A$, $f(4) = G$, $f(2) = H$, $f(3) = B$, $f(8) = C$, $f(7) = I$, $f(5) = J$, $f(6) = D$.

2. (5 points) The degree sequence of a graph is the list of the degrees of all the nodes in the graph, arranged in numerical order, largest to smallest. Is it possible to construct a (simple) graph with degree sequence: 4, 3, 3, 2, 0? Show how or briefly explain why this isn't possible.

Solution: This isn't possible. Since one of the five nodes has degree 4, it's connected to all the other nodes. But then we can't have a node with degree 0.

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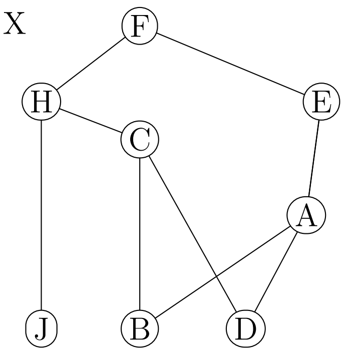
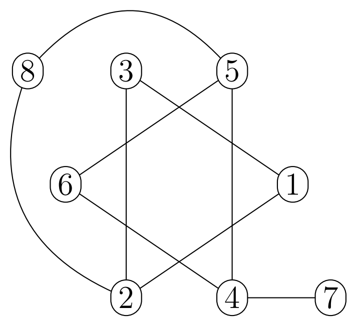


Graph Y

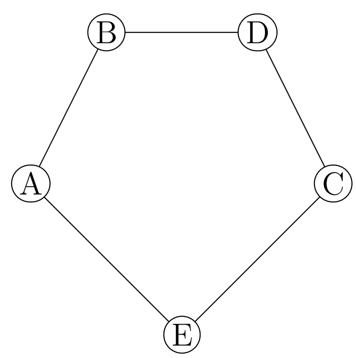


Solution: No, they are not isomorphic. Graph Y contains two 3-cycles but Graph X doesn't contain any 3-cycles.

2. (5 points) Suppose that $d(u, v)$ is the distance between nodes u and v (i.e. along the shortest path. Agent K claims that $d(u, v) + d(v, w) = d(u, w)$ for any nodes u, v, and w. Is he correct? Briefly explain why or give a counter-example.

Solution:

Agent K is wrong. Consider the graph to the right. $d(A, C) = 2$. $d(C, B)$ is also 2. So $d(A, C) + d(C, B) = 4$. But $d(A, B) = 1$. So $d(u, v) + d(v, w) \neq d(u, w)$ for these three nodes.



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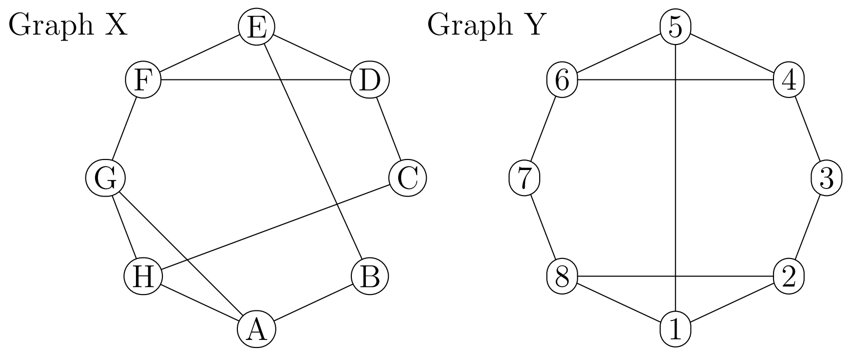

Solution: Yes, these graphs are isomorphic. Notice that the pairs of degree-2 nodes (B and C, 3 and 7) must be matched in one order or the other. Also notice that edge FG is distinctive because it connects the two triangles, so it must match 15 (in one direction or the other).

One possible map is f(C) = 7, f(B) = 3 for the degree-2 nodes. Then f(D) = 6, f(F) = 5, f(E) = 4 for one triangle. And f(H) = 8, f(A) = 2, and f(G) = 1 for the other triangle.

2. (5 points) The degree sequence of a graph is the list of the degrees of all the nodes in the graph, arranged in numerical order, largest to smallest. Is it possible to construct a (simple) graph with degree sequence: 4, 3, 3, 2, 2, 1? Show how or briefly explain why this isn't possible.

Solution: This isn't possible. By the Handshaking Theorem, the degrees have to add up to an even number. But these degrees add up to 15.

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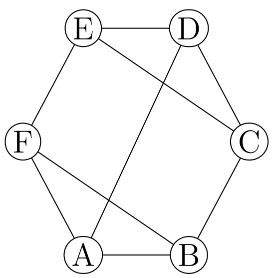

Solution: The triangle CDE can map onto itself or onto the triangle ABF (2 choices).

Having made that choice, we can permute the three nodes in CDE ($3! = 6$ choices). This determines the matches for the three nodes in the other triangle.

So there are $2 \cdot 3! = 12$ isomorphisms from the graph to itself.

2. (5 points) How many edges are in the complete bipartite graph $K_{10,5}$?

Solution: $10 \times 5 = 50$