CS173 Lecture 2: Propositional Logic

Gul Agha

Presented by Karl Palmskog

August 25, 2016

CS173 Announcements

If you aren't registered for the class:
Please write your name on the list at front!

CS173 Announcements

Lecture B website now up:
`https://courses.engr.illinois.edu/cs173/fa2016/B-lecture/`

Has:

- Slides for lectures
- Study problems
- Logistics information
- Moodle and Piazza links
- ...

CS173 Announcements

- "indus", without the quotes, is the password for self-enrollment on Moodle in the B lecture (for those students not automatically enrolled)
- There is not much at Moodle yet but you should check it next Tuesday for self-study assignments (autograded quizzes)
- If you registered for the course, you should either already have access to the course Piazza or should have received an invite to Piazza
- If you registered late, or if you want to register a different email address, let your TA or Professor Agha know

CS173 Announcements

Discussions sections start **this week**:

- Thursday A discussions, 1111 Siebel Center
- Friday A discussions, 1302 Siebel Center
- Friday B discussions, 1111 Siebel Center

Go to the section you signed up for. Attendance will be taken!

Goals of this lecture

- Become familiar with logic terminology
- Be able to translate between English and propositional logic expressions
- Be able to make truth tables and show whether statements are equivalent
- Be able to manipulate expressions to derive new statements

Propositions

- Statements that have a true/false value.

- Examples of propositions
  - I am male.
  - 34 is less than 20.
  - If the sky is blue, then the clouds are yellow.
- Examples of non-propositions
  - $x > 5$
  - Do your homework.
  - Is the sky blue?

Logical Operators and Truth Tables

- T, F, AND (^), OR (v), xor, not (¬)
  - On overhead
- Important:
  - "x AND y" is true only if both x and y are true
  - "x OR y" is true if x is true, if y is true, or if both x and y are true
- Examples: $p \wedge q, p \vee q, p \vee T, p \wedge F, p\ xor\ q, not\ p$
  - True or false? I am 33 or 97. I am 33 and 97. I voted for Obama or Romney.

overhead (2.1,2.2)

Implication

- Implies
  - x implies y, $x \rightarrow y$
  - If x, then y
  - When x is true, then y is always true.

- Examples
  - If it is a dog, then it can walk.
  - Every time I look at you, I go blind.

- Important: if the hypothesis is false, then the implication is always true
  - "If false, then false" is a true statement

- Bi-implies: $x \leftrightarrow y$
  - Both statements must be true (or false) at the same time

- Chaining implications: if $x \rightarrow y$ and $y \rightarrow z$, then $x \rightarrow z$

overhead (2.3)

Truth Tables for $\wedge$, $\vee$, $\neg$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	$\neg p$
T	F
F	T

Truth Tables for $\rightarrow$, $\leftrightarrow$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth Table for Complex Proposition

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T			
T	F			
F	T			
F	F			

Truth Table for Complex Proposition

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	
T	F	F	T	
F	T	F	T	
F	F	F	F	

Truth Table for Complex Proposition

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Contrapositive, converse, negation

Proposition:

"If the sky is green, then I'm a monkey's uncle."

- Converse
  - If I'm a monkey's uncle, then the sky is green.
- Contrapositive
  - If I'm not a monkey's uncle, then the sky is not green.
- Negation
  - The sky is green, but I am not a monkey's uncle.

overhead

Converse of $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$q \to p$
T	T	T
T	F	T
F	T	F
F	F	T

Contrapositive of $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\neg q$	$\neg p$	$\neg q \to \neg p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

Negation of $p \to q$

p	q	$p \to q$	$\neg(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

Manipulation and equivalence

- Logical Equivalence: two propositions are equivalent if they have the same T/F values for all cases
- DeMorgan's Laws
- Distributive and commutative laws
- $a \to b \equiv \neg a \vee b$

overhead (2.4)

Equivalence of $p \to q$ and $\neg p \vee q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

List of Useful Equivalences

$$
\begin{aligned}
\neg\neg p &\equiv p \\
\neg(p \wedge q) &\equiv \neg p \vee \neg q \\
\neg(p \vee q) &\equiv \neg p \wedge \neg q \\
\neg(p \rightarrow q) &\equiv p \wedge \neg q \\
p \rightarrow q &\equiv \neg p \vee q \\
p \vee (q \wedge r) &\equiv (p \vee q) \wedge (p \vee r) \\
p \wedge (q \vee r) &\equiv (p \wedge q) \vee (p \wedge r)
\end{aligned}
$$

Proving a Tautology by Manipulation

Have: $((p \vee q) \wedge \neg p) \rightarrow q$

$$
\begin{aligned}
\neg(((p \vee q) \wedge \neg p) \rightarrow q) &\equiv \\
((p \vee q) \wedge \neg p) \wedge \neg q &\equiv \\
(\neg p \wedge (p \vee q)) \wedge \neg q &\equiv \\
((\neg p \wedge p) \vee (\neg p \wedge q)) \wedge \neg q &\equiv \\
(F \vee (\neg p \wedge q)) \wedge \neg q &\equiv \\
(\neg p \wedge q) \wedge \neg q &\equiv \\
\neg q \wedge (\neg p \wedge q) &\equiv \\
(\neg q \wedge \neg p) \wedge (\neg q \wedge q) &\equiv \\
(\neg q \wedge \neg p) \wedge F &\equiv \\
F &
\end{aligned}
$$

1. Promise breakers are untrustworthy.
(If the person is a promise-breaker, then she is untrustworthy).

2. Wine drinkers are very communicative.
(If the person is a wine-drinker, then she is very communicative.)

3. A person who keeps her promises is honest.
(If the person keeps her promises, then she is trustworthy.)

4. No teetotalers are pawnbrokers.
(If the person does not drink wine, then she is not a pawnbroker.)

5. One can always trust a very communicative person.
(If the person is very communicative, then you can trust her.)

More Caroll logic puzzles: http://www.math.hawaii.edu/~hile/math100/logice.htm

overhead

Real-world Problem: Textual Entailment

Textual entailment: given a snippet of text *S*, does it entail a proposition *T*

(ENGLAND, June, 1989) - Christopher Robin is alive and well. He lives in England. He is the same person that you read about in the book Winnie the Pooh. As a boy, Chris lived in a pretty home called Cotchfield Farm. When Chris was three years old, his father wrote a poem about him. The poem was printed in a magazine for others to read. Mr. Robin then wrote a book. He made up a fairy tale land where Chris lived. His friends were animals. There was a bear called Winnie the Pooh. There was also an owl and a young pig, called a piglet. All the animals were stuffed toys that Chris owned. Mr. Robin made them come to life with his words. The places in the story were all near Cotchfield Farm. Winnie the Pooh was written in 1925. Children still love to read about Christopher Robin and his animal friends. Most people don't know he is a real person. He has written books of his own that tell what it is like to be famous. [REMEDIA]

1. Christopher Robin was born in England.
2. Winnie the Pooh is a title of a book.
3. Christopher Robin's dad was a magician.
4. Christopher Robin must be at least 65 now.

Example from Dan Roth

Predicate logic

- Predicate: propositions that have input variables with a range of values
  - For some integer x, $x > 10$
  - Cars that are red and speeding are likely to be ticketed: $isred(x) \wedge speeding(x) \rightarrow likely\_to\_be\_ticketed(x)$
  - A person's mother's mother is his/her grandmother

    $For\ every\ set\ of\ people\ x, y, z,$

    $mother(x, y) \wedge mother(y, z) \rightarrow grandmother(x, z)$

Quantifiers

- For some x: $\exists x$
  - Some creatures are greyhounds that run fast.
  - $\exists x$ in integers, $x^2 = 0$
  - $\exists x$ in reals, $x^2 > 0$

- For all x: $\forall x$
  - For all creatures, if it is fat, then it does not run well.
  - $\forall x$ in reals, $x^2 \geq 0$

- For exactly one x: $\exists!\, x$
  - There is exactly one fat creature than runs well.
  - $\exists!\, x$ in integers, $x^2 = 0$

Binding and scope

binding

$$\forall x, p(x) \rightarrow q(x)$$

$$\exists x \text{ in reals}, x^2 = 0$$

scope

Manipulating quantifiers

- Negation

$$\neg\left(\forall x, p(x)\right) \equiv \exists x, \neg p(x)$$
$$\neg\left(\exists x, p(x)\right) \equiv \forall x, \neg p(x)$$

  – "Not all dogs are fat" is equivalent to "At least one dog is not fat."
  – "There does not exist one fat dog" is equivalent to "All dogs are not fat."

- Contrapositive

$$\forall x, p(x) \rightarrow q(x) \equiv \forall x, \neg q(x) \rightarrow \neg p(x)$$

Examples of textual entailment

- Proposition: Jane lives in the United States
  1. Jane lives in New York City.
  2. If location x is contained in location y, then anyone who lives in x also lives in y.
  3. New York City is within the United States.

- Logic puzzle: What else can we infer?
  1. No fat creatures run well.
  2. Some greyhounds run well.

Entailment example

1. Jane lives in New York City.
2. If location x is contained in location y, then anyone who lives in x also lives in y.
3. New York City is within the United States.

Q: Does Jane definitely live in the US?

overhead

Entailment example

1. No fat creatures run well.
2. Some greyhounds run well.

Q: What else can we infer?

overhead

Quantifiers with two variables

- For all integers a and b, $a + b \geq a$ (false)

$\forall a \in Z, \forall b \in Z, a + b \geq a$ or $\forall a, b \in Z, a + b \geq a$

- For every real a, there exists an integer b, such that $a + b \geq a$ (true)

$$\forall a \in R, \exists b \in Z, a + b \geq a$$