

# CS 173: Fall 2014, A lecture

## Long-form homework 2

For this homework, you will submit a solution to one of the following problems via moodle. See moodle for which problem you have been assigned. It is due at 11:45pm on Friday, October 10th.

You must do this problem by yourself. You may not work with classmates.

The four problems are challenging in rather different ways.

The grading rubric will place a heavy emphasis on style and logical order. When the instructions for a problem specify a particular proof technique, you must use that method even if the proof could have been done in other ways. If you're solving a word problem, your solution must relate the basic mathematics (e.g. equations) back to the original topic of the problem (e.g. dishes at a restaurant).

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### Problem 1

Twenty-three mathematicians are eating dinner at Tang Dynasty and they have arranged to be seated at a special, unusually-large circular table. The table has a lazy Susan (central rotating circular tray) in the middle. Each person has ordered a different dish and (rather mysteriously) they all refuse to share.

The dishes of food are brought out and placed on the lazy Susan, one dish in front of every person. However, they are entirely mismatched, so each person has another person's dish. Prove that there is a way to rotate the lazy Susan so that at least two people have the correct dish that they ordered in front of them.

### Problem 2

For any integers  $s$  and  $t$ , let's define  $L(s, t) = \{sx + ty \mid x, y \in \mathbb{Z}\}$ .

Prove the following claim.

Claim: For any integers  $a, r, m$ , where  $r$  is positive, if  $a \equiv m \pmod{r}$ , then  $L(a, m) \subseteq L(r, m)$ .

Your proof must directly use the textbook definitions of congruence and divisibility.

### Problem 3

Tireless Tyrion has designed an app to let you type the High Valyrian language more quickly on a standard 3x4 phone key pad, as long as you stick to words in his 40,000-word Valyrian dictionary. You can write each word by typing its first five characters, hitting the corresponding key only once for each character (not several times, as with normal texting interfaces). For example, to write “margaret” you would push the button sequence 62742. If the word is shorter than 5 characters, zero is used to fill it up to five numbers. E.g. to write “kor”, you would push the button sequence 56700. Fortunately, for text messaging purposes all words in Valyrian can be written reasonably well using the standard 26 lowercase alphabetic characters.

Even without knowing anything about the specific words used in Valyrian, what’s wrong with this design from a discrete math point of view?

### Problem 4

Define a relation  $\sim$  on the set of all functions from  $\mathbb{R}$  to  $\mathbb{R}$  by the rule  $f \sim g$  if and only if there is a  $k \in \mathbb{R}$  such that  $f(x) = g(x)$  for every  $x \geq k$ . Prove that  $\sim$  is an equivalence relation.