CS 173: Fall 2014, A lecture
Long-form homework 1

For this homework, you will do one of the problems below. See moodle for which one you have been assigned to do. It is due 11:45pm on Friday, September 19th.

Your homework will be submitted by typing directly into submission boxes on moodle. See the homework web page for pointers to information on style and equation formatting.

You must do this problem by yourself, perhaps with some help from the course staff. You may not work with classmates. It must be in your own words, with your own formatting.

The grading rubric will place a heavy emphasis on style and logical order. Be sure to use your best mathematical English, use good formatting and punctuation, put your steps in logical order, use connector words, introduce variables at the start of the proof, and so forth.

When the instructions for a problem specify a particular proof technique, you must use that method. Even if the proof can be done in other ways, we would like you to practice using this approach.

Problem 1

For any two real numbers x and y, the harmonic mean of x and y is $H(x, y) = \frac{2xy}{x+y}$. This is a form of averaging that penalizes the case when either of the inputs is very small, often used for combining two performance numbers when evaluating a computer program.

(a) This definition has a small but important bug. What is it?

(b) The more familiar arithmetic mean is $M(x, y) = \frac{x+y}{2}$. When is $H(x, y)$ equal to $M(x, y)$?

(c) Prove that your answer to (b) is correct.

Problem 2

If p and r are the precision and recall of a test, then the F1 measure of the test is defined to be

$$F(p, r) = \frac{2pr}{p + r}$$

Prove that, for all positive reals p, r, and t, if $t \geq r$ then $F(p, t) \geq F(p, r)$.

Problem 3

A point (x, y) in $\mathbb{R}^2$ is on the unit circle if and only if $x^2 + y^2 = 1$. Let's define an operation $\triangle$ on 2D points by

$$(x, y) \triangle (s, t) = (xs - yt, xt + ys)$$

Prove the following claim using a direct proof.

> Claim: For any points p and q in $\mathbb{R}^2$, if p and q are on the unit circle, then $p \triangle q$ is on the unit circle.

Problem 4

Prove the following claim:

> Claim: For any integer x, y, s, t, m, n, where $s \neq 0$, if $xs = yt$ and $ms = nt$, then $xn = ym$.

Your proof must break into cases, depending on whether t is zero or non-zero. Your proof must not use fractions (though you can cancel non-zero factors that occur on both sides of an equation).