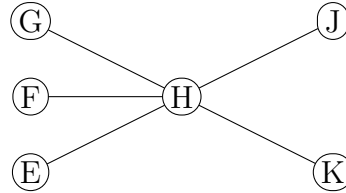
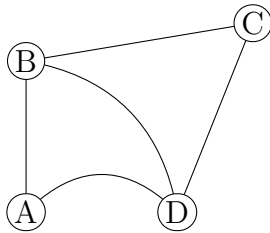


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Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2



1. (11 points) How many isomorphisms are there from G (above) to itself? Justify your answer and/or show your work clearly .

Solution: Nodes A and B can be swapped (2 choices). Nodes B and D can be swapped (2 choices). And nodes E, F, G, K, and J can be permuted ($5!$ choices). H must match to itself.

Since the above choices are all independent, there are $4 \cdot 5! = 480$ isomorphisms total.

2. (4 points) Is G bipartite? Briefly explain why or why not.

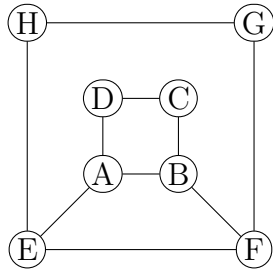
Solution: No, it is not bipartite. The lefthand part has a 3-cycle, which isn't possible in a bipartite graph.

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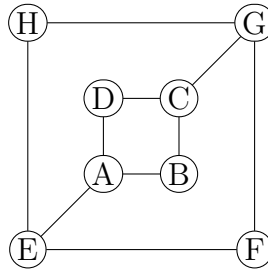
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Graph X



Graph Y



1. (11 points) Are graphs X and Y (above) isomorphic? Justify your answer.

Solution: No, they are not isomorphic. In graph X, the degree-2 nodes are connected to each other in pairs (e.g. G to H). In graph Y, each degree-2 node is connected only to degree-3 nodes.

2. (4 points) Complete this statement of the Handshaking Theorem.

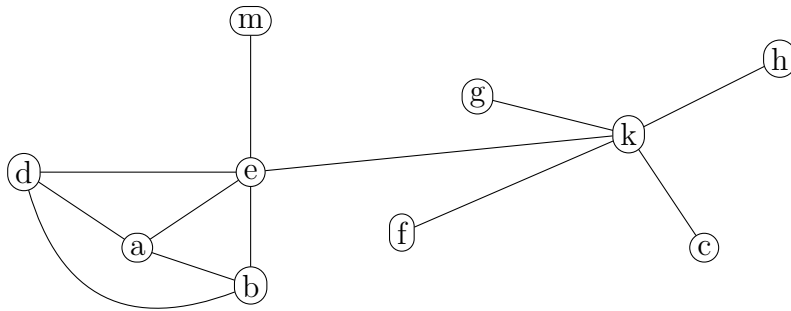
For any graph G with set of nodes V and set of edges E, ...

Solution: the sum of the degrees of the nodes in V is equal to twice the number of edges in E. (Or the same thing written out as an equation.)

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1. (11 points) How many isomorphisms are there from G (above) to itself? Justify your answer and/or show your work clearly .

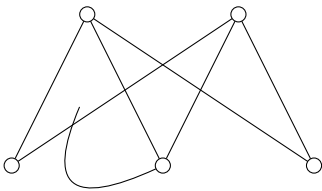
Solution: E and K are the only degree-5 nodes. But they can't be swapped because K is connected only to degree-1 nodes and E is connected to some nodes of higher degree. So E and K have to match to themselves.

Given that, M must match to itself (only degree-1 node connected to E).

Nodes A, B and D can be permuted ($3!$ choices). Nodes C, F, G, and H can be permuted ($4!$ choices). So there are $3! \cdot 4! = 144$ isomorphisms in total.

2. (4 points) Draw a picture of the graph $K_{2,3}$.

Solution:

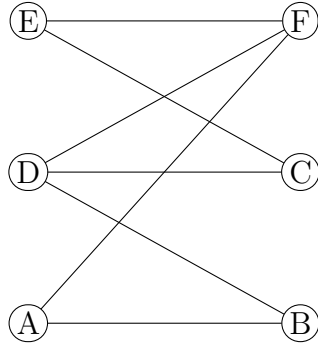


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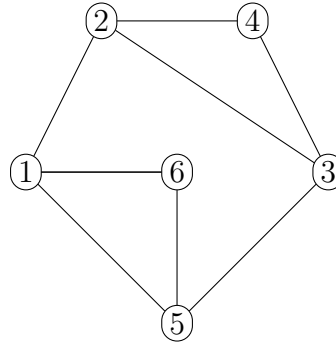
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Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

Graph X



Graph Y



1. (11 points) Are graphs X and Y (above) isomorphic? Justify your answer.

Solution: No, they are not isomorphic. Graph X has 7 edges but graph Y has 8 edges. [And various other features fail to match as well.]

2. (4 points) The complete graph K_7 contains 7 vertices. How many edges does it have?

Solution: It has $\frac{7 \cdot 6}{2} = 21$ edges.