Examlet 3, Part A

NETID:

FIRST:	LAST:

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

1. (4 points) $A = \{\text{fox}, \text{cat}\}$ $B = \{3, 4\}$ $C = \{3, 7\}$

   $A \times (B \cap C) = A \times \{3\} = \{(fox, 3), (cat, 3)\}$

   $A \cap B = \emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   $\emptyset \in A$ — true for all sets A ☐ true for some sets A ☑ false for all sets A ☐

   If $x \in A \cap B$, then $x \in A$. — true for all sets A and B ☑ true for some sets A and B ☐ false for all sets A and B ☐

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[6]^6 + [5]^3$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[6]^2 = [36] = [3]$

   $[6]^6 = [3]^3 = [27] = [5]$

   $[5]^3 = [125] = [4]$

   $[6]^6 + [5]^3 = [5] + [4] = [9]$

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1. (4 points) $A = \{\text{fox, tiger, wolf, eagle, cat}\}$ $B = \{3, 4\}$ $C = \{6, 7\}$

   $A \times (B \cap C) = A \times \emptyset = \emptyset$

   $| A \times (B \cup C) | = 20$

   Because $B \cup C$ contains 4 elements and A contains 5 elements.

2. (4 points) Check the (single) box that best characterizes each item.

   $A \cap (B \cup C)$ $= (A \cap B) \cup (A \cap C)$ true for all sets A [√] true for some sets A [ ] false for all sets A [ ]

   $\emptyset$ is an element of $\mathbb{Z}$ [ ] a subset of $\mathbb{Z}$ [√] both [ ] neither [ ]

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[8]^{22}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[8]^2 = [64] = 9$

   $[8]^4 = [9]^2 = [81] = [4]$

   $[8]^8 = [4]^2 = [16] = [5]$

   $[8]^{16} = [5]^2 = [3]$

   $[8]^{22} = [8]^{16} \cdot [8]^4 \cdot [8]^2 = [3][4][9]$

   $[3][4][9] = [3][36] = [3][3] = [9]$

   So $[8]^{22} = [9]$

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1. (4 points) $A = \{\text{fox}, \text{cat}\}$ $B = \{3, 4\}$

   $A \cap B = \emptyset$

   $\{p + q^2 \mid 0 \leq p \leq 2 \text{ and } 1 \leq q \leq 2\} =$

   [Correction announced at exam: p and q are integers.]

   $\{1, 2, 3, 4, 5, 6\}$

2. (4 points) Check the (single) box that best characterizes each item.

   $\{13, 14, 15\} \times \emptyset =$ $\emptyset$ [√] $\{\emptyset\}$ [ ] $\{13, 14, 15\}$ [ ]

   $|A \cup B| = |A| + |B|$ true for all sets A [ ] true for some sets A [√]
   false for all sets A [ ]

3. (7 points) In $\mathbb{Z}_9$, find the value of $[5]^{21}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 8$.

   Solution:

   $[5]^2 = [25] = [7]$

   $[5]^4 = [7]^2 = [49] = [4]$

   $[5]^8 = [4]^2 = [16] = [7]$

   $[5]^{16} = [7]^2 = [49] = [4]$

   $[5]^{21} = [5]^{16} \cdot [5]^4 \cdot [5] = [4][4][5] = [80] = [8]$

Examlet 3, Part A

NETID:

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Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

1. (4 points) $A = \{4, 5, 9\}$ $B = \{\text{arya}, \text{bran}\}$ $C = \{\ (4, 5),\ (4, 9)\ \}$

   $B \times A = \{\ (\text{arya}, 4), (\text{arya}, 5), (\text{arya}, 9), (\text{bran}, 4), (\text{bran}, 5), (\text{bran}, 9)\}$

   $A \cap C = \emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   $A \times B = B \times A$

   True for all sets A and B ☐ False for all sets A and B ☐

   True for some sets A and B ☑

   $\emptyset \subseteq A$

   true for all sets A ☑ true for some sets A ☐

   false for all sets A ☐

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{19}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10]$

   $[7]^4 = [100] = [9]$

   $[7]^8 = [9]^2 = [81] = [3]$

   $[7]^{16} = [3]^2 = [9]$

   $[7]^{19} = [7]^{16} \cdot [7]^{[}2] \cdot [7] = [9][10][7]$

   $[9][10][7] = [90][7] = [-1][7] = [-7] = [6]$

   So $[7]^{19} = [6]$