CS 173, Fall 2014
Examlet 12, Part B

NETID:

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Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(a) (9 points) Suppose that A is a set and P is a collection of subsets of A. Using precise language and/or notation, state the conditions P must satisfy to be a partition of A.

Solution: P cannot contain the empty set. Every element of A must belong to exactly one element of P.

The second condition is frequently split into two sepaate conditions. That is, every element of A must belong some element of P, and two distinct elements of P cannot overlap.

(b) (6 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{R} \to \mathbb{P}(\mathbb{Z})$ then $f(17)$ is

- an integer ☐
- a set of integers ☑ $\surd$
- one or more integers ☐
- a power set ☐

Pascal's identity states that $\binom{n}{k}$ is equal to

- $\binom{n-1}{k} + \binom{n-1}{k-1}$ ☑ $\surd$
- $\binom{n-1}{k} + \binom{n-1}{k+1}$ ☐
- $\binom{n-1}{k} + \binom{n-2}{k}$ ☐

$\{\{a, b\}, c\} = \{a, b, c\}$

- True ☐
- False ☑ $\surd$

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Let $f : \mathbb{R}^2 \to \mathbb{P}(\mathbb{R}^2)$ be defined by $f(x, y) = \{(p, q) \in \mathbb{R}^2 \mid x^2 + y^2 = p^2 + q^2\}$
Let $T = \{f(x, y) \mid (x, y) \in \mathbb{R}^2\}$.

(a) (6 points) Answer the following questions:

$f(0, 0) = \{(0, 0)\}$

Describe (at a high level) the elements of $f(0, 36)$: the circle centered on the origin with radius 6.

The cardinality of (aka the number of elements in) T is: infinite

(b) (7 points) Is T a partition of $\mathbb{R}^2$? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: Yes. The output of f is never the empty set. None of these circles (plus the dot at the origin) overlaps any of the others. And jointly they cover all of the plane.

(c) (2 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \emptyset$ — true for all sets ☐ — true if $A \cap B = \emptyset$ ☐ — false for all sets ☑ $\surd$

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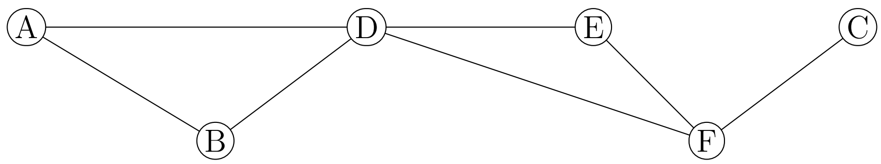
Graph G is at right.
V is the set of nodes in G.



Define $f : V \to \mathbb{P}(V)$ by $f(p) = \{n \in V \ : \ \deg(n) < \deg(p)\}$, where deg(n) is the degree of node n.
Let $P = \{f(p) \mid p \in V\}$.

(a) (6 points) Fill in the following values:

f(A) = $\{C\}$

f(C) = $\emptyset$

P = $\{\emptyset, \{C\}\{A, B, C, E\}, \{A, B, C, E, F\}\}$

(b) (7 points) Is P a partition of V? For each of the conditions required to be a partition, briefly explain why P does or doesn't satisfy that condition.

Solution: No. There is partial overlap between elements of P, P contains the empty set and node D is not in any of the elements of P (all bad).

(b) (2 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(\emptyset)$ $\emptyset$ ☐ $\{\emptyset\}$ ☑ $\{\{\emptyset\}\}$ ☐ $\{\emptyset, \{\emptyset\}\}$ ☐

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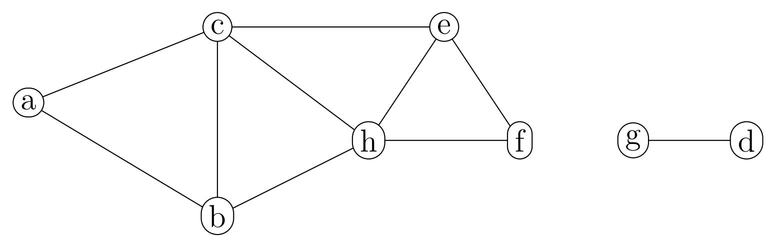
Graph G is at right.
V is the set of nodes.
E is the set of edges.
ab (or ba) is the edge between a and b.



Let $f : V \to \mathbb{P}(E)$ be defined by $f(n) = \{e \in E \mid n \text{ is an endpoint of } e\}$. And let $T = \{f(n) \mid n \in V\}$.

(a) (6 points) Fill in the following values:

$|E| = 10$

$f(d) = \{gd\}$

$f(h) = \{bh, ch, eh, fh\}$

(b) (7 points) Is T a partition of E? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: T is not a partition of E. T does not contain the empty set (good) and each edge in E is in some member of T (good). However, each edge in E is in two members of T, so there is partial overlap among the members of T (bad).

(c) (2 points) Check the (single) box that best characterizes each item.

For any integers n and k ($n \geq k \geq 0$), $\binom{n}{k} = \binom{n}{n-k}$. True [√] False [ ]