CS 173, Fall 2014
Examlet 12 Part A

NETID:

FIRST:	**LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(a) (9 points) Use proof by contradiction to show that $\sqrt{7} \geq 1 + \sqrt{2}$.

Solution: Suppose not. That is, suppose that $\sqrt{7} < 1 + \sqrt{2}$.

Since the numbers are all positive, we can square both sides to get $7 < (1 + \sqrt{2})^2 = 1 + 2\sqrt{2} + 2$. So then $4 < 2\sqrt{2}$. So $2 < \sqrt{2}$. Squaring both sides again, we get $4 < 2$ which is clearly false.

Since its negation led to a contradiction, our original claim must have been true.

(b) (6 points) Justin needs to pick 17 toy animals to give to children at a party. The animals come in 5 kinds: dogs, dinosaurs, cows, lizards, and fish. How many different ways can he choose his set of toys?

Solution: $\binom{17+4}{4}$.

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(a) (9 points) Recall that nodes in a full binary tree have either zero or two children. Suppose we are building a full binary tree with unlabelled nodes whose leaves are all at levels k or $k+1$, with p leaves at level $k+1$. How many different ways can we construct such a tree? Briefly justify your answer.

Solution: Notice that nodes in each level come in pairs, because the tree is full. So p must be even. Let $p = 2n$.

The only thing we get to choose when constructing these trees is which nodes at level k have two children (vs. zero children). There are 2^k nodes at level k, of which n will have children. So we have $\binom{2^k}{n}$ options for constructing the tree.

(b) (6 points) State the negation of the following claim, in English (not shorthand notation), moving all negations (e.g. "not") so that they are on individual predicates.

For every dinosaur k, if k is blue, then k is not vegetarian or k is friendly.

Solution: There is a dinosaur k such that k is blue but k is vegetarian and k is not friendly.

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(a) (9 points) Use proof by contradiction to show that $\log_5 2$ is irrational.

Solution: Suppose not. That is, suppose that $\log_5 2$ is rational. Then $\log_5 2 = \frac{a}{b}$, where a and b are integers, b non-zero.

Raising 5 to the power of both sides, we get $2 = 5^{\frac{a}{b}}$. Raising both sides to the bth power, we get $2^b = 5^a$. Since 2 and 5 are both prime, this equation can hold only if $a = b = 0$. But we know that b is non-zero. So we have a contradiction.

Since its negation led to a contradiction, our original claim must have been true.

(b) (6 points) In the polynomial $(2x - 3y)^{20}$, what is the coefficient of the term x^5y^{15}? (Please do not attempt to simplify your formula.)

Solution: According to the Binomial Theorem, this term is $\binom{20}{5}(2x)^5(-3y)^{15}$.

This is equal to $\binom{20}{5}2^5x^5(-3)^{15}y^{15}$.

So the coefficient on this term is $\binom{20}{5}2^5(-3)^{15}$.

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(a) (9 points) Use proof by contradiction to show that, for any integer n, at least one of the three integers n, $2n + 1$, $4n + 3$ is not divisible by 7.

Solution: Suppose not. That is, suppose that n, $2n + 1$, $4n + 3$ are all divisible by 7. Then their sum $n + (2n + 1) + (4n + 3)$ must be divisible by 7. So $7n + 4$ must be divisible by 7. But then 4 would need to be divisible by 7, which isn't true.

Since its negation led to a contradiction, our original claim must have been true.

(b) (6 points) Margaret's home is defended from zombies by wallnuts, peashooters, and starfruit. At each timestep, she can make one move, which adds or deletes one plant from her arsenal. If she starts with 3 wallnuts, 2 peashooters, and 19 starfruit, how many different sequences of 25 moves will get her to a configuration with 7 wallnuts, 13 peashooters, and 9 starfruit?

Solution: The sequence of 25 moves needs to add 4 wallnuts, add 11 peashooters, and delete 10 starfruit. So we need to pick 4 moves in the sequence to be the ones that add wallnuts, and then 11 of the remaining 21 moves to be ones that add peashooters. So our number of choices is

$$\binom{25}{4}\binom{21}{11}$$