CS 173, Fall 2014
Examlet 10 Part A

NETID:

FIRST: | **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(15 points) Use (strong) induction to prove the following claim:

Claim: For all integers $n \geq 2$, $(2n)! > 2^n n!$

Base Case(s): At $n = 2$, $(2n)! = 4! = 24$. $2^n n! = 4 \cdot 2 = 8$. So $(2n)! > 2^n n!$

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Suppose that $(2n)! > 2^n n!$ for all $n = 2, 3, \ldots, k$ for some integer $k \geq 2$.

Inductive Step: Notice that $2k + 1 \geq 1$ because k is positive. And $(2k)! > 2^k k!$ by the induction hypothesis.

So then

$(2(k+1))! = (2k+2)(2k+1)(2k)! \geq (2k+2)(2k)! > (2k+2)(2^k k!) = (k+1)2^{k+1}k! = 2^{k+1}(k+1)!$.

So $(2(k+1))! > 2^{k+1}(k+1)!$ which is what we needed to show.

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(15 points) Use (strong) induction to prove the following claim:

Claim: For any natural number n and any real number $x > -1$, $(1+x)^n \geq 1 + nx$.

Base Case(s): At $n = 0$, $(1+x)^n = (1+x)^0 = 1$ and $1 + nx = 1 + 0 = 1$. So $(1+x)^n \geq 1 + nx$.

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Suppose that $(1+x)^n \geq 1 + nx$ for any natural number $n \leq k$, where k is a natural number.

Inductive Step: By the inductive hypothesis $(1+x)^k \geq 1 + kx$. Notice that $(1+x)$ is positive since $x > -1$. So $(1+x)^{k+1} \geq (1+x)(1+kx)$.

But $(1+x)(1+kx) = 1 + x + kx + kx^2 = 1 + (1+k)x + kx^2$.

And $1 + (1+k)x + kx^2 \geq 1 + (1+k)x$ because kx^2 is non-negative.

So $(1+x)^{k+1} \geq (1+x)(1+kx) \geq 1 + (1+k)x$, and therefore $(1+x)^{k+1} \geq 1 + (1+k)x$, which is what we needed to show.

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(15 points) Recall the following fact about real numbers

Triangle Inequality: For any real numbers x and y, $|x + y| \leq |x| + |y|$.

Use this fact and (strong) induction to prove the following claim:

Claim: For any real numbers $x_1, x_2, \ldots, x_n$ ($n \geq 2$), $|x_1 + x_2 + \ldots + x_n| \leq |x_1| + |x_2| + \ldots + |x_n|$.

Base Case(s): At $n = 2$, the claim is exactly the Triangle Inequality, which we're assuming to hold.

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Suppose that $|x_1 + x_2 + \ldots + x_n| \leq |x_1| + |x_2| + \ldots + |x_n|$ for any list of n real numbers $x_1, x_2, \ldots, x_n$, where $2 \leq n \leq k$.

Inductive Step: Let $x_1, x_2, \ldots, x_{k+1}$ be a list of $k + 1$ real numbers.

Using the Triangle Inequality, we get

$|x_1 + x_2 + \ldots + x_k + x_{k+1}| = |(x_1 + x_2 + \ldots + x_k) + x_{k+1}| \leq |(x_1 + x_2 + \ldots + x_k)| + |x_{k+1}|$

But, by the inductive hypothesis $|(x_1 + x_2 + \ldots + x_k)| + |x_{k+1}| \leq |x_1| + |x_2| + \ldots + |x_k| \leq |x_{k+1}|$.

Putting these two equations together, we get

$|x_1 + x_2 + \ldots + x_k + x_{k+1}| = |(x_1 + x_2 + \ldots + x_k) + x_{k+1}| \leq (|x_1| + |x_2| + \ldots + |x_k|) + |x_{k+1}|$.

So $|x_1 + x_2 + \ldots + x_k + x_{k+1}| \leq |x_1| + |x_2| + \ldots + |x_k| + |x_{k+1}|$, which is what we needed to show.

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(15 points) Use (strong) induction to prove the following claim:

Claim: For any positive integer n, $\sum_{p=1}^{n} \frac{1}{\sqrt{p}} \geq \sqrt{n}$

You may use the fact that $\sqrt{n+1} \geq \sqrt{n}$ for any natural number n.

Base Case(s): At $n = 1$, $\sum_{p=1}^{n} \frac{1}{\sqrt{p}} = 1$ Also $\sqrt{n} = 1$. So $\sum_{p=1}^{n} \frac{1}{\sqrt{p}} \geq \sqrt{n}$.

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Suppose that $\sum_{p=1}^{n} \frac{1}{\sqrt{p}} \geq \sqrt{n}$ for $n = 1, 2, \ldots, k$, for some integer $k \geq 1$.

Inductive Step: $\sum_{p=1}^{k} \frac{1}{\sqrt{p}} \geq \sqrt{k}$ by the inductive hypothesis.

So

$$\sum_{p=1}^{k+1} \frac{1}{\sqrt{p}} = \frac{1}{\sqrt{k+1}} + \sum_{p=1}^{k} \frac{1}{\sqrt{p}} \geq \frac{1}{\sqrt{k+1}} + \sqrt{k} = \frac{1+\sqrt{k}\sqrt{k+1}}{\sqrt{k+1}} \geq \frac{1+\sqrt{k}\sqrt{k}}{\sqrt{k+1}} = \frac{1+k}{\sqrt{k+1}} = \sqrt{k+1}$$

So $\sum_{p=1}^{k+1} \frac{1}{\sqrt{p}} \geq \sqrt{k+1}$, which is what we needed to show.