State Diagrams

Lecture 24

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- The number of possible states could be finite or infinite (for e.g. if the system remembers the sequence of inputs seen so far, or even just the number of inputs so far)

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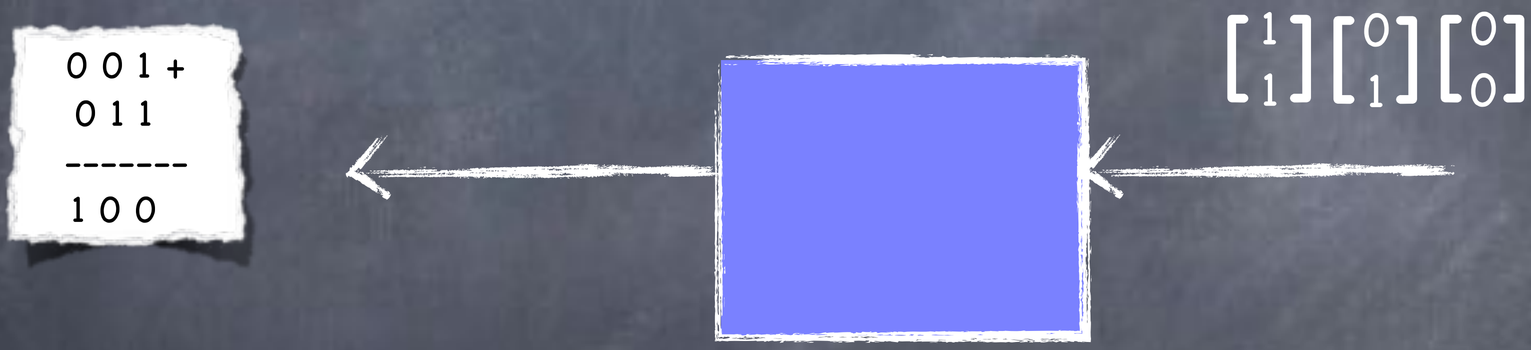

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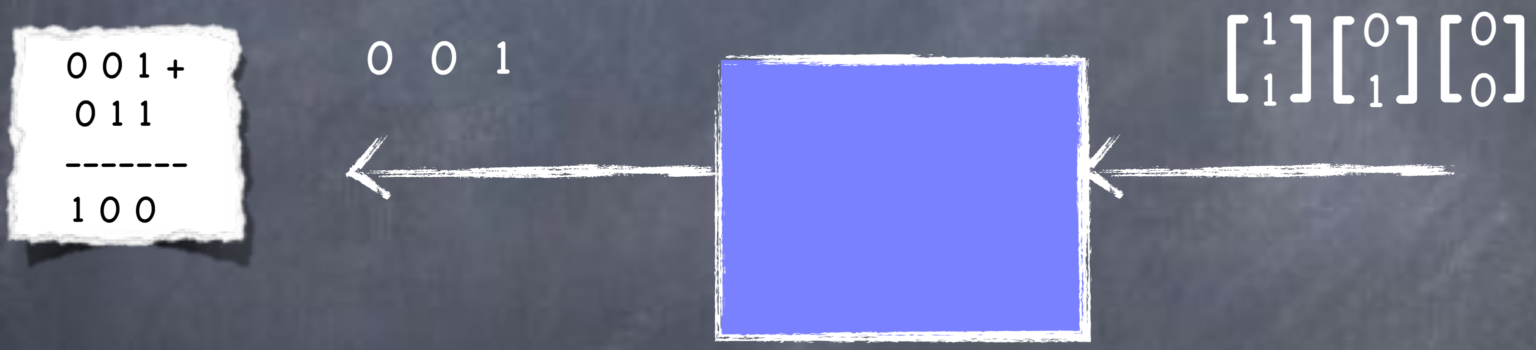

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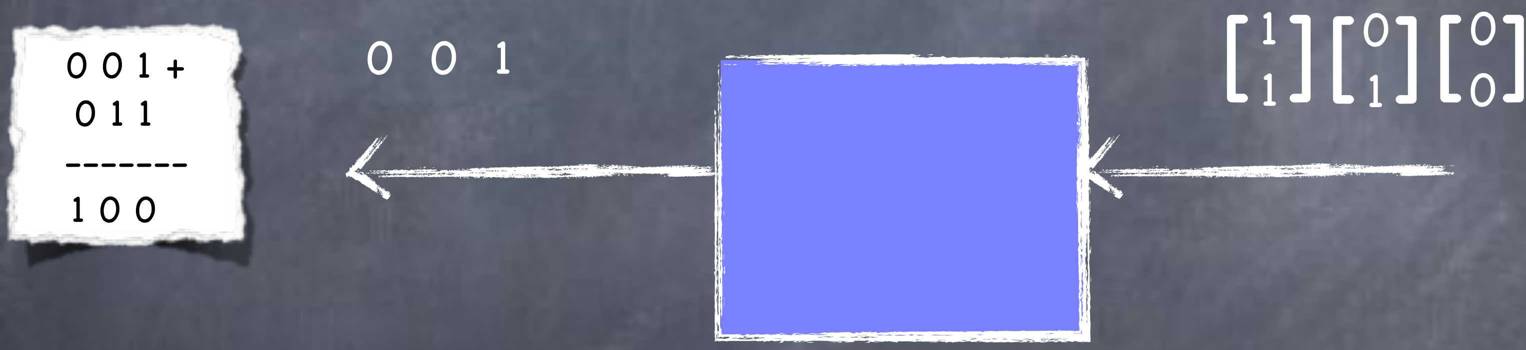

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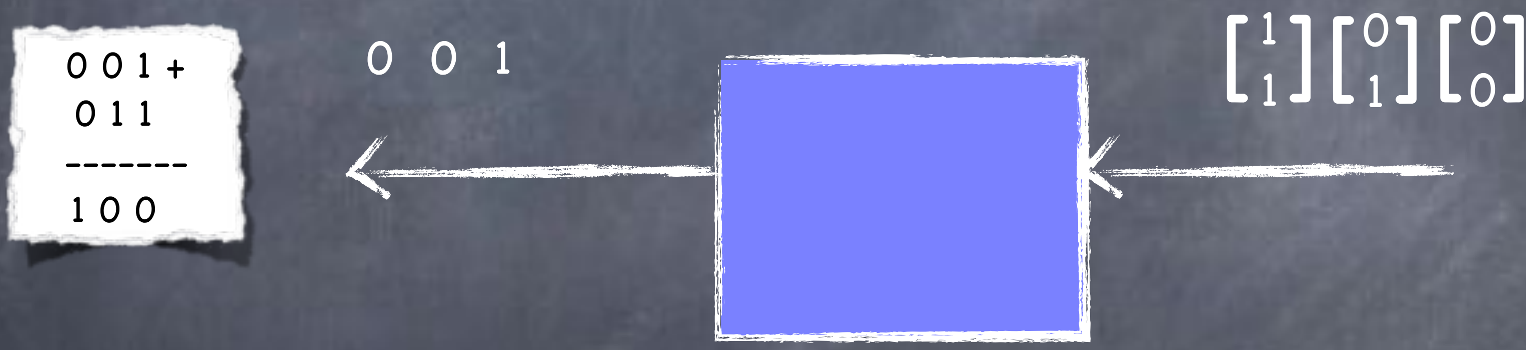

- What should the system remember?
  - The "carry": a single bit
  - State diagram has two nodes

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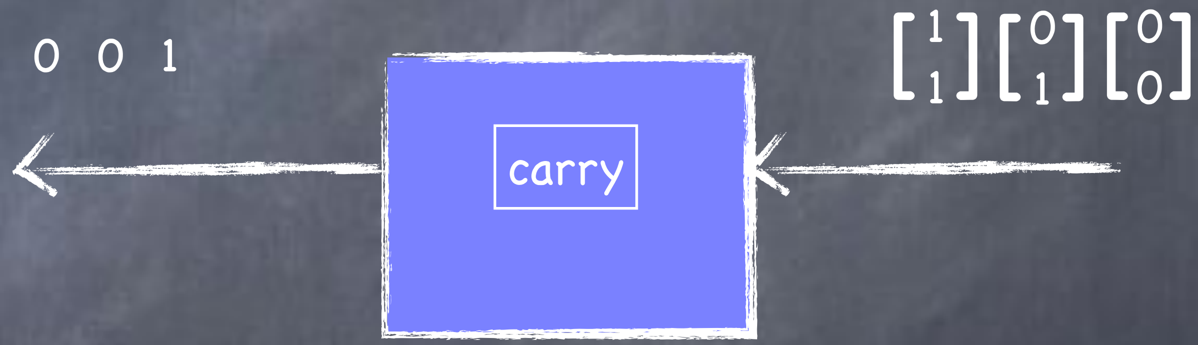

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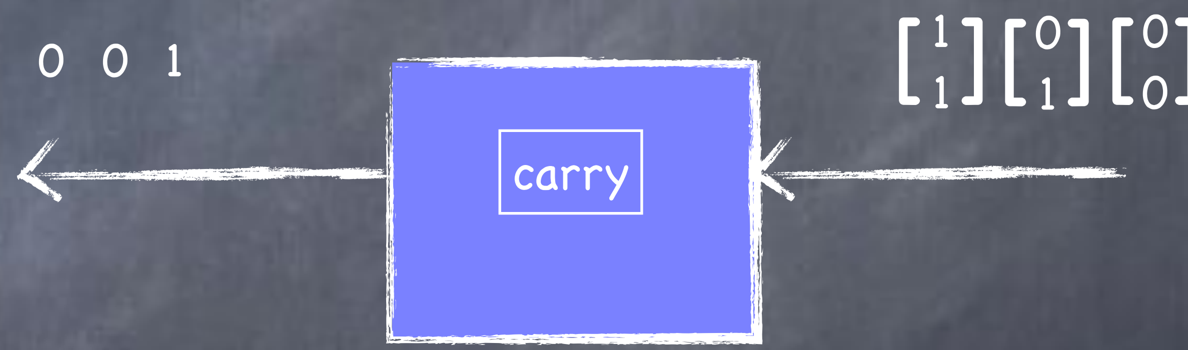

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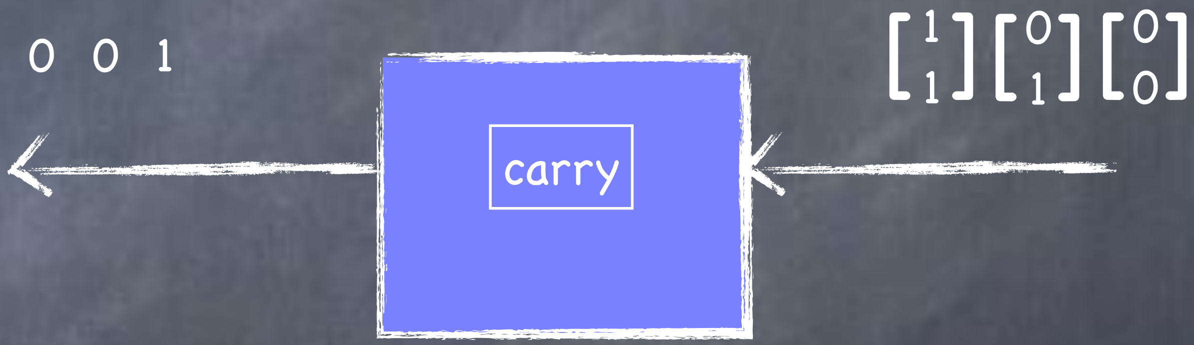

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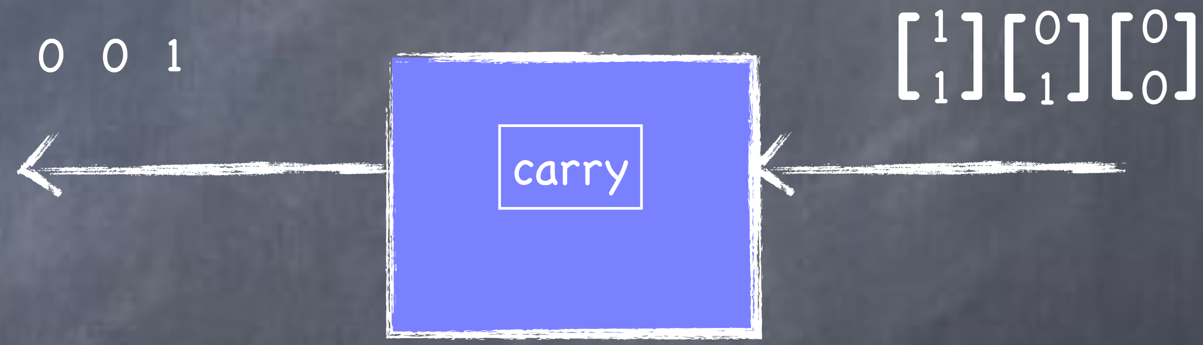

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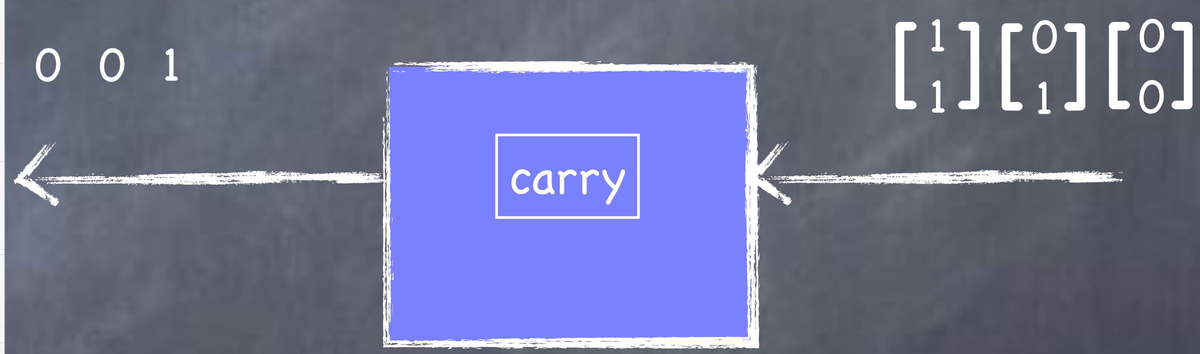

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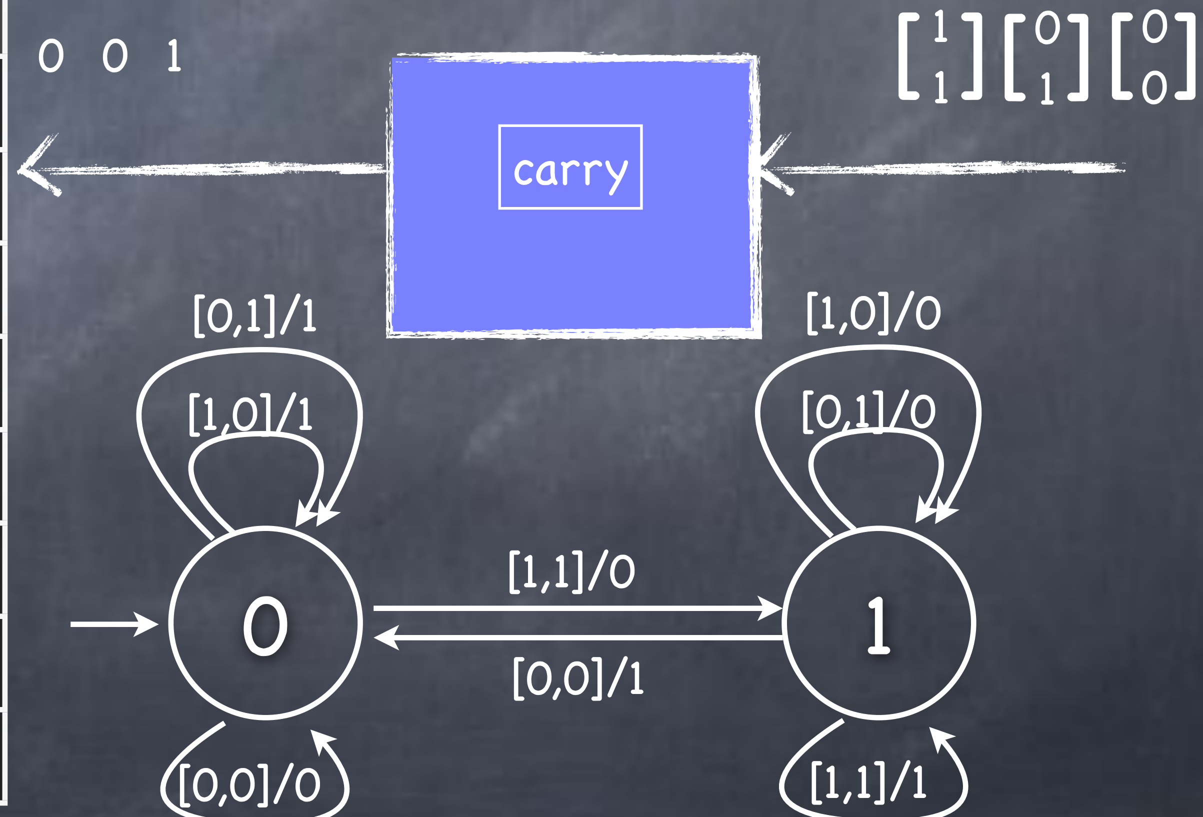

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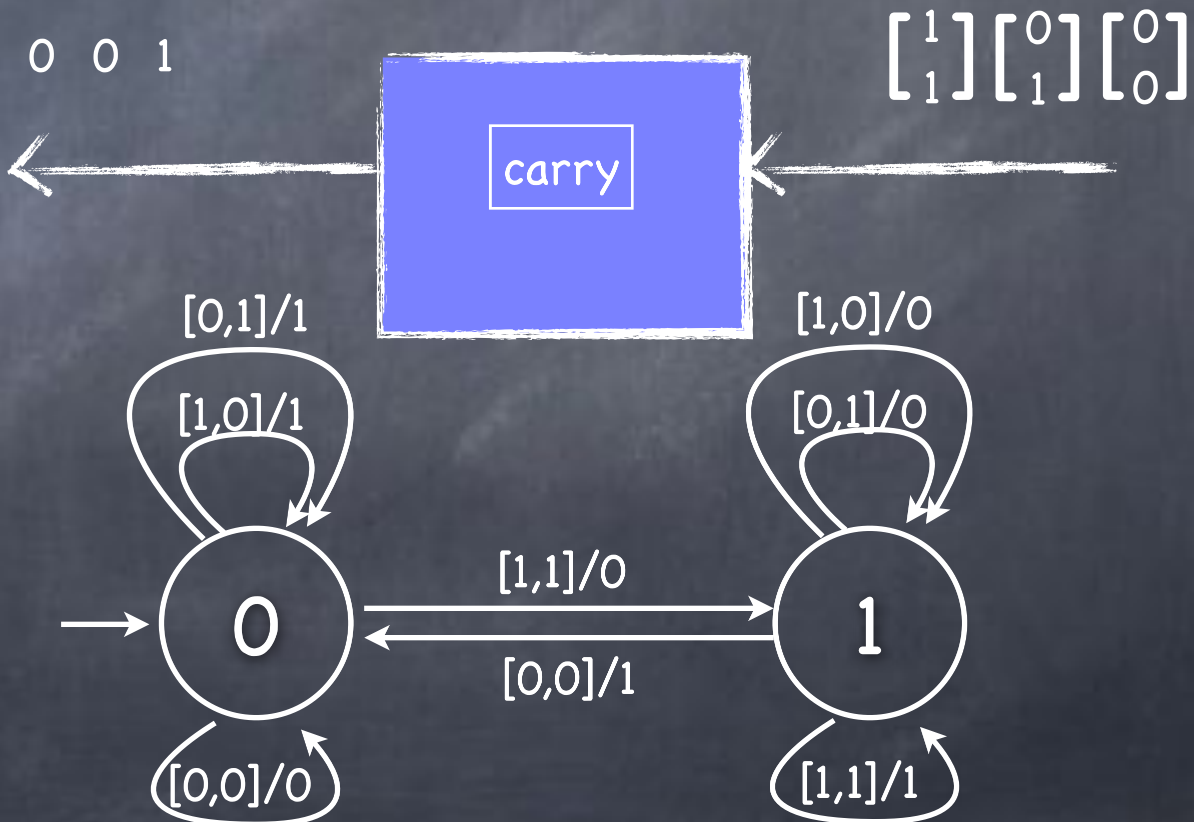

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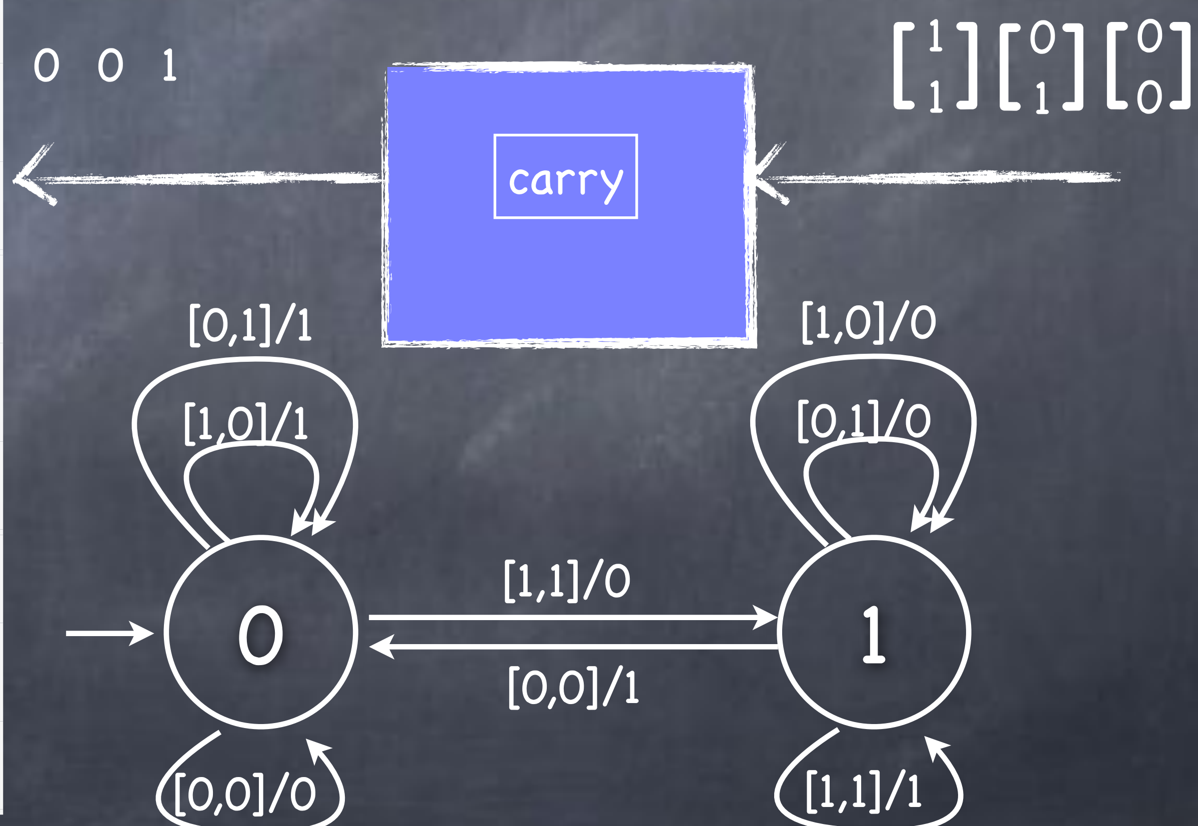

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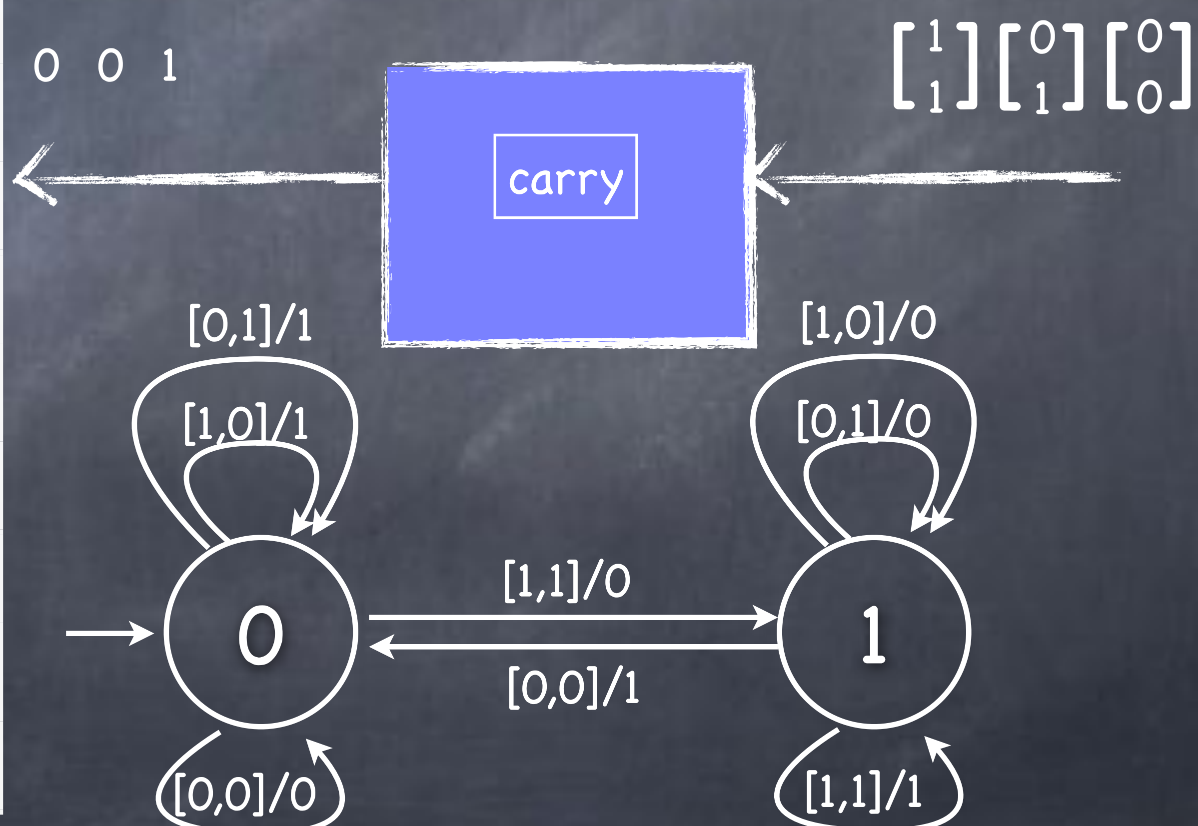

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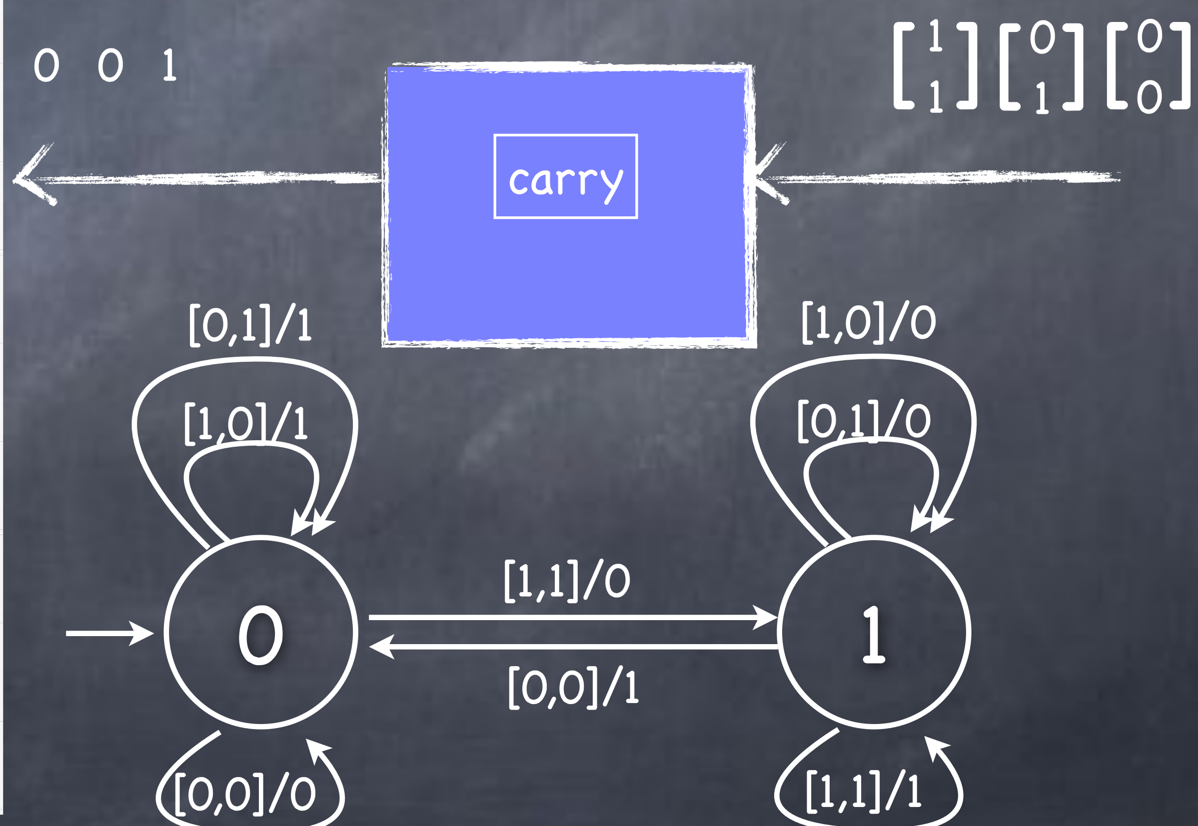

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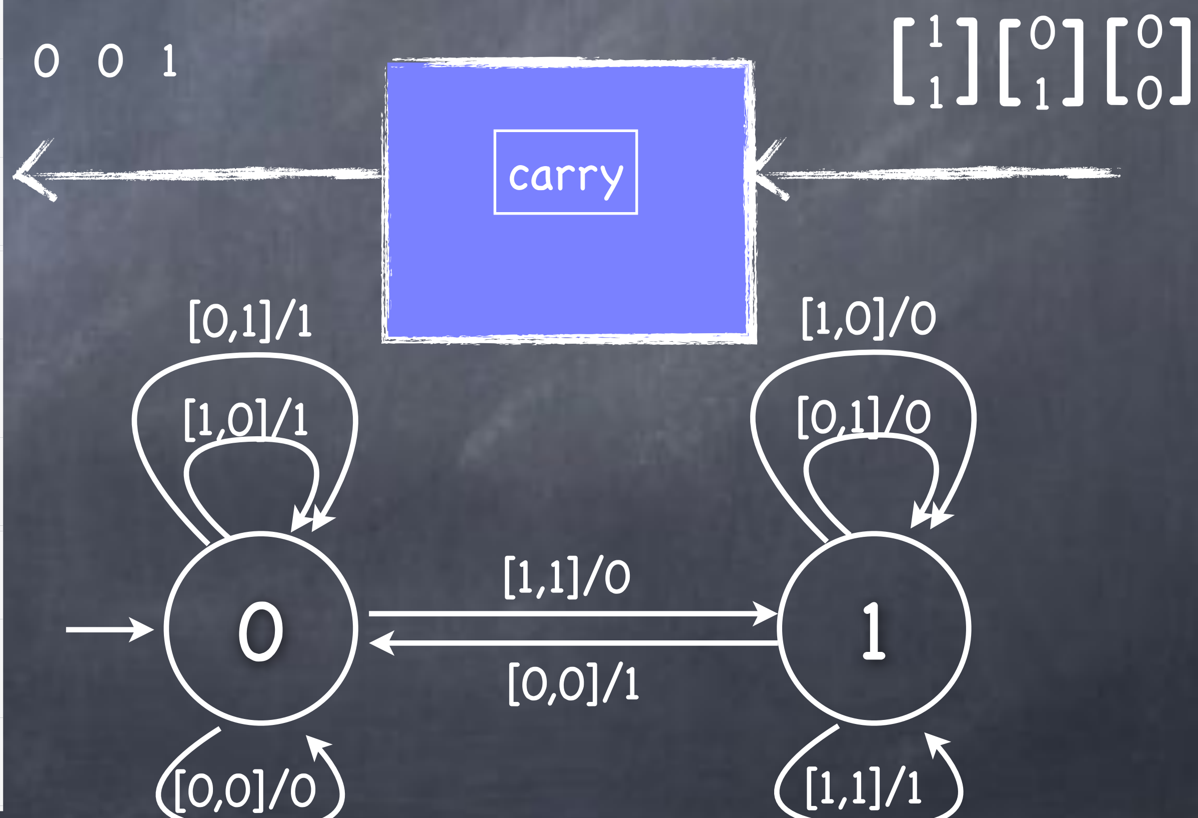

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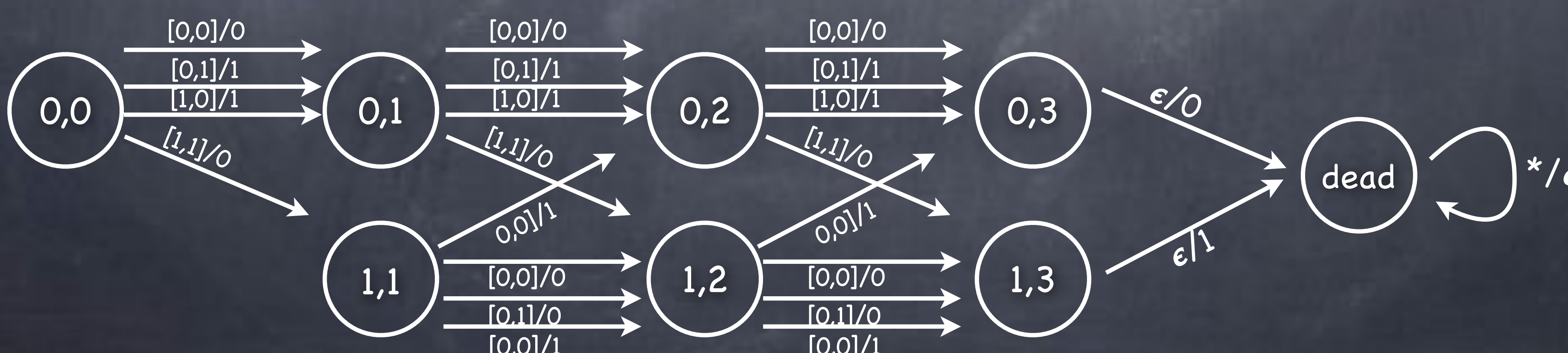

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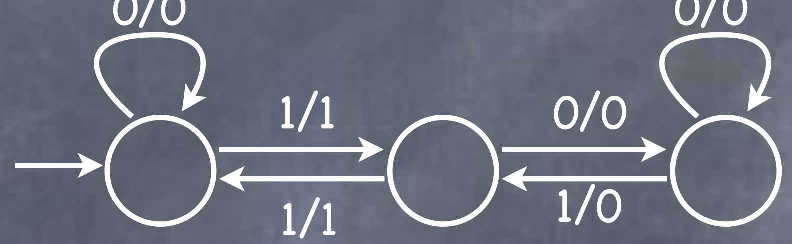

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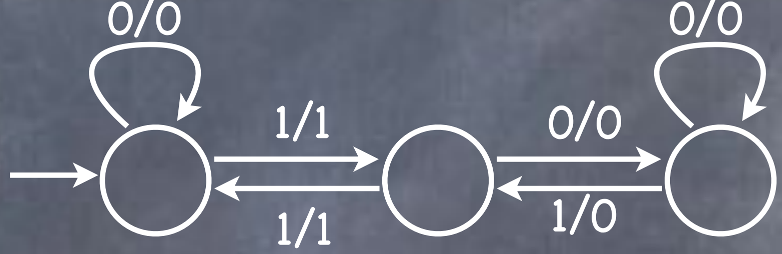

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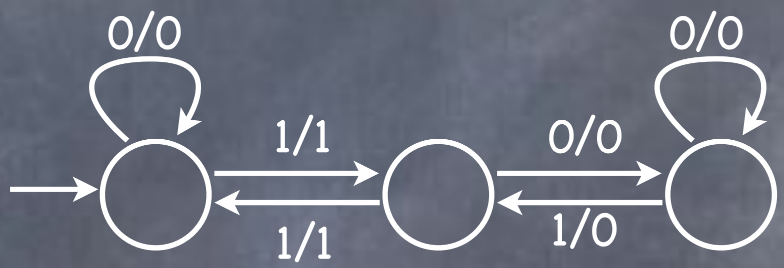

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(0*11)* **10** 0* **1** (0|1)*

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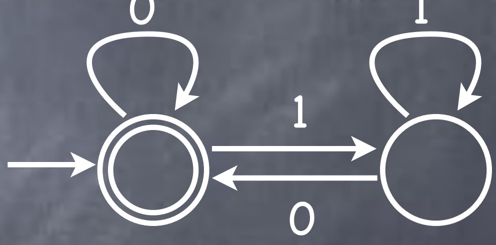

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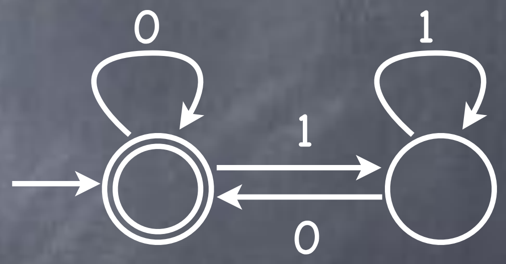

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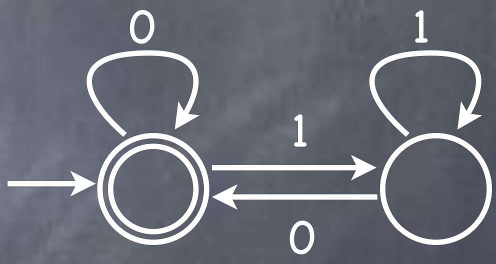

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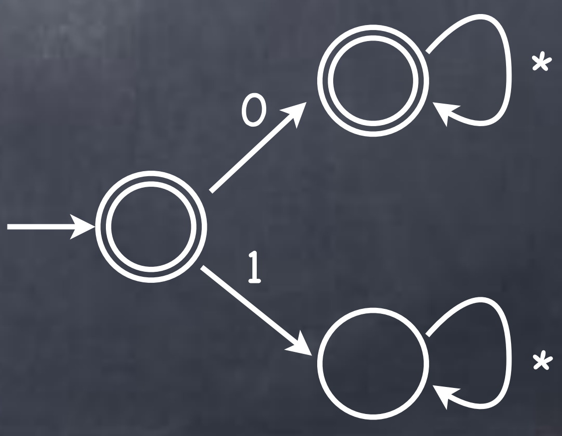

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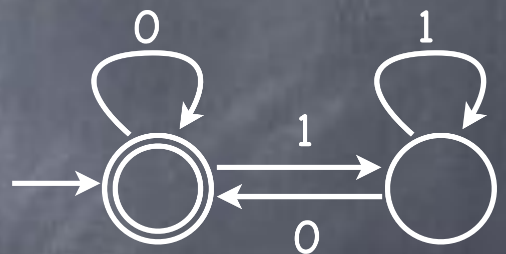

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- How about deciding if the number is a multiple of say 5?

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- How many states must an acceptor for multiples of 5 have, when the inputs are given as binary digits of a non-negative number, with MSB first?
(Treat empty input as number 0.)

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  B. 4
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Need to only remember x (mod 5), where x is the number seen so far. Next number x′ is 2x or 2x+1 depending on the current input bit. x′ (mod 5) is determined by x (mod 5)

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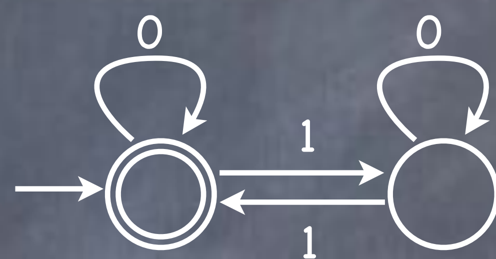

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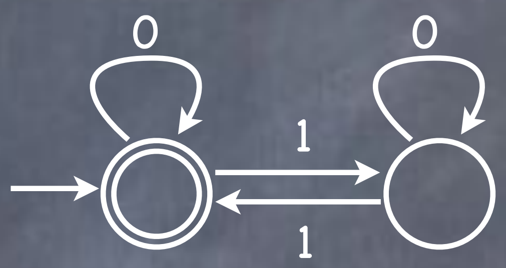

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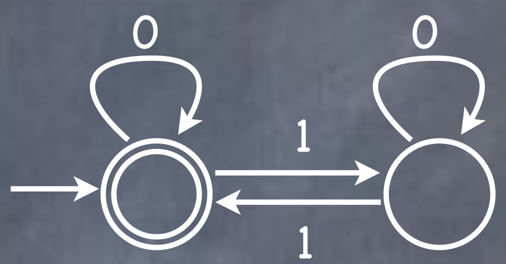

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Odd number of 1s

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(T,T,Bob)
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    - Later (in CS173).

Non-determinism

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- Sometimes probabilistic machine: Non-deterministic machine + probabilities associated with the multiple transitions

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    - An appropriate data structure (sometimes a "hash table") can give (almost) the best of both worlds

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    - Later...