CS 173, Spring 2012

Midterm 2, April 3 2012

NAME:

NETID (e.g. alincoln16, not 123987654):

DISCUSSION DAY:

DISCUSSION TIME:

Problem	1	2	3	4	5	6	Total
Possible	12	9	6	6	7	10	50
Score							

We will be checking photo ID's during the exam. Have your ID handy.
(Forgot your ID? See us at the end of the exam.)

Turn in your exam at the front when you are done.

You have 75 minutes to finish the exam.

Problem 1: Multiple Choice and True/False (12 points)

Check the box that <u>best</u> characterizes each item. Check only one box per statement. If you change your answer, make sure its easy to tell which box is your final selection.

If a graph has an Euler circuit, then all nodes must have even degree.

true ☐   **false** ☐

The number of leaves in a binary tree of height h is at most 2^h.

true ☐   **false** ☐

$$\sum_{k=1}^{n} \frac{1}{2^k} = 1 - \frac{1}{2^n}$$

true ☐   **false** ☐

Two graphs are isomorphic if there is a bijection between their nodes.

true ☐   **false** ☐

$g : \mathbb{Z} \to \mathbb{Z}, \ g(x) = 7 - \left\lfloor \frac{x}{3} \right\rfloor$

| yes onto yes 1-to-1 ☐ | yes onto not 1-to-1 ☐ | not onto yes 1-to-1 ☐ | not onto not 1-to-1 ☐ |

$f : \mathbb{Z} \to \mathbb{Z}, \ f(x) = x + 3$ if x is even, and $f(x) = x - 22$ if x is odd

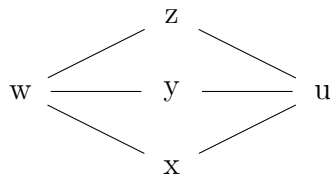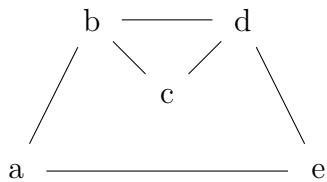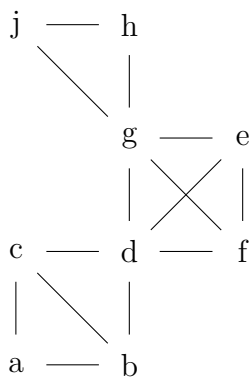| yes onto yes 1-to-1 ☐ | yes onto not 1-to-1 ☐ | not onto yes 1-to-1 ☐ | not onto not 1-to-1 ☐ |

Problem 2: Short Answer (9 points)

(a) (4 points) Are these two graphs isomorphic? Justify your answer.

b —— d
c
a ———— e

z
w —— y —— u
x

(b) (5 points) In the following graph, how many different paths are there from c to f? Remember that a path cannot repeat vertices. Show your work.

j —— h
g —— e
c —— d —— f
a —— b

Problem 3: Recursion Trees (6 points)

Consider the function defined by:

$$f(1) = 13$$

$$f(n) = 5f\left(\tfrac{n}{6}\right) + n^3, \text{ for } n \geq 2$$

(a) What is an appropriate domain for this function, i.e. for which input values does this definition provide a well-defined output value? (For parts (b)-(d), you should assume all inputs are from this set.)

(b) The height h of the recursion tree for this function is:

(c) The sum of all the values at level k (where $k < h$) of the tree is:

(d) Give the sum of all the leaf values. You do not need to "simplify" your formula.

Problem 4: Graphs (6 points)

Recall that if G is a graph, then $\chi(G)$ is its chromatic number. Suppose that G and H are each connected graphs but H is not connected to G. Suppose also that G and H each have at least two nodes and at least one edge. Dr. Evil merges G and H into a single graph T as follows. He picks two adjacent vertices a and b from G, and also two adjacent vertices c and d from H. He adds an edge connecting a and c, and merges b and d into a single vertex.

For example, if G and H are as shown on the left, then T might look as shown on the right.

1. How is $\chi(T)$ related to $\chi(G)$ and $\chi(H)$? (Be sure to address any special cases.)

2. Justify your answer:

Problem 5: Tree Induction (7 points)

Use strong induction to prove that a full 5-ary tree of height h has at least $4h + 1$ leaves. (Recall that a m-ary tree is full if each node has either zero or m children.)

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

Write your netID, in case this page gets pulled off:

Problem 6: Induction (10 points)

Let the function $f : \mathbb{N} \to \mathbb{R}$ be defined by

$$f(0) = \frac{2}{3}$$

$$f(1) = \frac{8}{9}$$

$$f(n) = \frac{4}{3}f(n-1) - \frac{1}{3}f(n-2), \quad \text{for } n \geq 2$$

Use strong induction on n to prove that $f(n) = 1 - \frac{1}{3^{n+1}}$ for any natural number n.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step: