CS 173, Spring 2012
Midterm 1, 23 February 2012

NAME:

NETID (e.g. bobama23, not 123987654):

DISCUSSION DAY:

DISCUSSION TIME:

You will lose a point if you don't accurately write the day and time of the discussion you are officially registered for. If you don't know the day and/or time of your discussion, you may consult the photo rosters at the podium before turning in your exam.
If you have recently changed section, check here: ☐

Problem	1	2	3	4	5	6	Total
Possible	5	10	8	9	9	9	50
Score							

We will be checking photo ID's during the exam. Have your ID handy.
(Forgot your ID? See us at the end of the exam.)

Turn in your exam at the front when you are done.

You have 75 minutes to finish the exam.

INSTRUCTIONS (read carefully)

- There are 6 problems, each on a separate page. Make sure you have a complete exam. The point value of each problem is indicated next to the problem, and on the cover page table.
- Points may be deducted for solutions which are correct, but hard to read, hard to understand, poorly explained, or excessively complicated.
- Brief explanations and/or showing work, even when not explicitly asked for, may increase partial credit for buggy answers. Partial credit for multiple-choice questions is very rare.
- It is not necessary to simplify or calculate out complex constant expressions such as $(0.7)^3(0.3)^5$, $\frac{0.15}{3.75}$, 3^{17}, and $7!$, unless it is explicitly indicated to completely simplify.
- It is wise to skim all problems and point values first, to best plan your time.
- This is a closed book exam.
  Turn off your cell phone now.
  No notes or electronic devices of any kind are allowed.
  These should be secured in your bag and out of reach during the exam.
- Do all work in the space provided, using the backs of sheets if necessary.
  If your work is on the backside then you must clearly indicate so on the problem.
  See the proctor if you need more paper.
- Please bring any apparent bugs or ambiguity to the attention of the proctors.
- After the midterm is over, you may discuss its contents with other students **only** after verifying that they have also taken the exam (e.g. they aren't about to take a makeup exam).

Problem 1: Multiple choice (5 points)

Check the most appropriate box for each statement. Check only one box per statement. If you change your answer, make sure it's easy to tell which box is your final selection.

(a) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, y \leq x$

True ☐ **False** ☐

(b) cardinality of $\{p + q \in \mathbb{N} \mid p \leq 2 \text{ and } q \leq 2\}$

3 ☐ 4 ☐ 5 ☐

9 ☐ infinite ☐

(c) $\lfloor \lceil x \rceil \rfloor = \lceil \lfloor x \rfloor \rceil$

true for any x in $\mathbb{R}$ ☐

false for any x in $\mathbb{R}$ ☐

true for some x in $\mathbb{R}$ ☐

(d) $\sum_{i=1}^{p-1} i =$

$\frac{p(p+1)}{2}$ ☐ $\frac{p(p-1)}{2}$ ☐

$\frac{(p-1)^2}{2}$ ☐ $\frac{(p-1)(p+1)}{2}$ ☐

(e) $\emptyset \subseteq A$

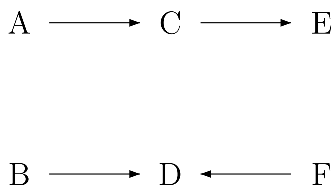
true for any set A ☐

false for any set A ☐

true for some sets A ☐

Problem 2: Short answer (10 points)

(a) (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$



Reflexive: ☐ **Irreflexive**: ☐

Symmetric: ☐ **Antisymmetric**: ☐

Transitive: ☐

(b) (3 points) Using precise mathematical words and notation, define what it means for a function $f : A \to B$ to be "one-to-one" (also called "injective"). Avoid words such as "unique" which are difficult to use in a fully-precise way.

(c) (2 points) What are the three properties that define an equivalence relation?

Problem 3: Sets and functions (8 points)

$$\begin{aligned} A &= \{\text{Ford}, \text{Tesla}, \text{Mazda}\} \\ B &= \{\text{sedan}, \text{wagon}, \text{pickup}, \text{hatchback}, \text{minivan}\} \\ C &= \{\text{GM}, (\text{Mazda}, \text{Volvo}), \text{Ford}\} \end{aligned}$$

(a) (3 points) List the elements of $B \times (A \cap C)$.

(b) (2 points) How many one-to-one functions $B \to (A \times C)$ are there?

(c) (3 points) Give a set D which is a subset of B and give a function $h : D \to A$ which is onto, but is not one-to one.

Problem 4: Number Theory (9 points)

(a) (3 points) In $\mathbb{Z}_{13}$, find the value of $([7])^{18}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

(b) (3 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

Claim: For all positive integers a, b, and c, if $\gcd(a, bc) > 1$, then $\gcd(a, b) > 1$ and $\gcd(a, c) > 1$.

(c) (3 points) Disprove the claim from part (b) using a concrete counter-example. Briefly explain why your counter-example works.

Problem 5: Relation proof (9 points)

Let $M = \{(3y + 1, y) \mid y \in \mathbb{R}^+\}$. That is, M is the collection of all all 2D points of the form $(3y + 1, y)$ in the first quadrant.

Let P be a relation on M defined by $(a, b)P(p, q)$ if and only if $aq \geq pb$.

Prove that P is antisymmetric. You must work directly from the definitions of M and P, using basic rules of algebra. Use your best mathematical style: proof in logical order, variables introduced, and key steps justified.

Write your netID, in case this page gets pulled off:

Problem 6: Number Theory Proof (9 points)

For any integers s and t define $L(s, t) = \{sx + ty \mid x, y \in \mathbb{Z}\}$.
Thus, $L(s, t)$ consists of all integers that can be expressed as the sum of multiples of s and t.

Prove the following claim. Your proof must use the definition of divisibility; you may not use lemmas about manipulating divides relationships. You must prove the set inclusion by choosing an element from the smaller set and showing that it is also a member of the larger set.

Claim: For any integers a, r, m, where r is positive, if $a \equiv m \pmod{r}$, then $L(a, m) \subseteq L(r, m)$.