# CS 173: Discrete Structures, Fall 2012
# Exam 1 Review

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These problems are to help you review for the first midterm. They should not be handed in.

1. **Set Operations**

   Suppose you were given the following sets:

$$
\begin{aligned}
\mathbf{A} &= \{\text{Piano}, (\text{Violin}, \text{Viola}, \text{Cello}), \text{Guitar}\} \\
\mathbf{B} &= \{(\text{Flute, Piccolo}), \text{Cymbals}\} \\
\mathbf{C} &= \{\text{Piano}, \text{Flute}\} \\
\mathbf{D} &= \{(\text{Violin, Viola, Cello}), (\text{Flute, Piccolo})\}
\end{aligned}
$$

   List the elements of the set or find the values for the following expressions:

   (a) $|A|$

   (b) $A \cup D$

   (c) $A \cap C$

   (d) $B \cap C$

   (e) $A - (B - C)$

   (f) $(B \cap D) \times C$

   (g) $A \times \emptyset$

   (h) $C \times \{\emptyset\}$

2. **Counting with sets**

   In our role-playing game, an evil character may be an elf or a troll, it may be red, green, brown, or black, and it may have scales or hair. A good character may be an elf or a human or a lion, it may be green, brown, or blue, and it has hair or fur. How many different character types do we have?

   Suppose we have a 26 character alphabet. How many 6-letter strings start with PRE or end with TH?

3. **Euclidean algorithm**

   Trace the execution of the Euclidean algorithm for computing GCD on the inputs $a = 837$ and $b = 2015$. That is, give a table showing the values of the main variables ($x$, $y$, $r$) for each pass through the loop. Explicitly indicate what the output value is.

4. **Direct Proof Using Congruence mod k**

   In the book, you will find several equivalent ways to define congruence mod k. For this problem, use the following definition: for any integers $x$ and $y$ and any positive integer $m$, $x \equiv y \pmod{m}$ if there is an integer $k$ such that $x = y + km$.

   Using this definition prove that, for all integers $a$, $b$, $c$, $p$, $q$ where $p$ and $q$ are positive, if $a \equiv b \pmod{p}$ and $c \equiv b \pmod{q}$ and $q|p$, then $a - 2c \equiv (-b) \pmod{q}$.

5. **Equivalence classes**

   Let $A = \mathbb{R}^{\geq 0} \times \mathbb{R}^{\geq 0} - \{(0,0)\}$, i.e. pairs of non-negative reals in which no more than one of the two numbers is zero.

   Consider the equivalence relation $\sim$ on $A$ defined by

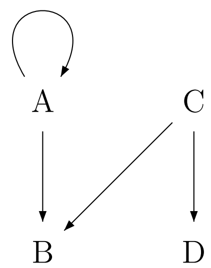
   $$(x,y) \sim (p,q) \quad \text{iff} \quad (xy)(p+q) = (pq)(x+y)$$

   (a) List four elements of $[(3,1)]$. Hint: what equation do you get if you set $(x,y)$ to $(3,1)$ and $q = 2p$?

   (b) Give two other distinct equivalence classes that are not equal to $[(3,1)]$.

   (c) Describe the members of $[(0,4)]$.

6. **Relation properties**




**Reflexive**: ☐ **Irreflexive**: ☐

**Symmetric**: ☐ **Antisymmetric**: ☐

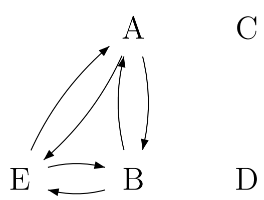
**Transitive**: ☐

$\sim$ is the relation on $\mathbb{R}$ such that $x \sim y$ if and only if $xy = 1$

**Reflexive**: ☐ **Irreflexive**: ☐

**Symmetric**: ☐ **Antisymmetric**: ☐

**Transitive**: ☐




**Reflexive**: ☐ **Irreflexive**: ☐

**Symmetric**: ☐ **Antisymmetric**: ☐

**Transitive**: ☐

7. **Proofs on Relations**

   (a) Define a relation $\sim$ on the set of all functions from $\mathbb{R}$ to $\mathbb{R}$ by the rule $f \sim g$ if and only if $\exists k \in \mathbb{R}$ such that $f(x) = g(x)$ for every $x \geq k$. Prove that $\sim$ is an equivalence relation.