CS 173: Discrete Structures, Fall 2009
Homework 7

This homework contains 5 problems worth a total of 42 regular points. It is due on Friday, October 23 at noon. Put your homework in the appropriate dropbox in the Siebel basement.

1. **Big-O proofs [8 points]**

   (a) Prove that $\frac{x^3+2x}{2x+1}$ is $O(x^2)$.

   (b) Use a proof by contradiction to show that 5^n is not $O(3^n)$.

2. **Unrolling [8 points]**

   Use unrolling to find a closed form for the function $f : \mathbb{N} \to \mathbb{R}$ defined by

   - $f(0) = 0$
   - $f(n) = 5f(n-1) + 1$ for all $n \geq 1$

   Show at least three steps of unrolling, show the unrolling pattern compactly using a summation, then convert to a simple algebraic equation. Hint: you may need to consult a table of closed-forms for familiar summations, e.g. the one in Rosen section 2.4.

3. **Recursion trees [8 points]**

   Consider the following recurrence:

   - $T(1) = d$
   - $T(n) = T(n/3) + T(2n/3) + cn$

   where c and d are constants. Let's analyze T when the input n is a power of 3.

   (a) Draw a recursion tree for T, in which each node shows the contribution of the non-recursive term cn.

   (b) How high is this tree (exact height, not just big-O)?

   (c) Use the tree to derive a closed-form solution for T, up to big-O.

   Hint: see lecture 22 for examples. In part (c), "up to big-O" means that your solution should have the form "$T(n)$ is $O(xxx)$" and, therefore, you can ignore low-order terms and constant multipliers in your solution.

4. **"Strong" induction [8 points]**

   Suppose that $f : \mathbb{Z}^+ \to \mathbb{Z}$ is defined by

   - $f(1) = 3$
   - $f(2) = 5$
   - $f(n) = 3f(n-1) - 2f(n-2)$ for all $n \geq 3$.

   Use strong induction to show that $f(n) = 2^n + 1$ for every positive integer n. Hint: you must use strong induction, because that's the main point of this problem.

5. **Induction with an inequality [10 points]**

   Use induction to prove that the following equation holds for all integers $n \geq 2$:

   $$\sum_{k=n+1}^{2n} \frac{1}{k} \geq \frac{7}{12}$$