CS 173: Discrete Structures, Fall 2009
Homework 2 Solutions (Corrected)

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1. **[8 points] Negating things**

   Give the negation of the following logical expressions, using logical equivalences to move the "not" operators onto the smallest elements possible. For example, to negate $\forall x, [P(x) \rightarrow Q(x)]$, we first negate the whole thing $\neg\forall x[P(x) \rightarrow Q(x)]$, then convert this to $\exists x[\neg(P(x) \rightarrow Q(x))]$, and finally to $\exists x[P(x) \wedge \neg Q(x)]$. (For simplicity, we've omitted the domains for the quantified variables.)

   (c) $\exists x[\neg(P(x) \wedge Q(x)) \vee (F(x) \rightarrow P(x)]$

   (d) $\forall z[(P(z) \wedge \neg M(z)) \rightarrow \neg P(z)]$

   Solution:

   (c) (2 points)
   $\neg(\exists x[\neg(P(x) \wedge Q(x)) \vee (F(x) \rightarrow P(x))]) \equiv$
   $\forall x[\neg(\neg(P(x) \wedge Q(x)) \vee (F(x) \rightarrow P(x)))] \equiv$
   $\forall x[(P(x) \wedge Q(x)) \wedge (F(x) \wedge \neg P(x))] \equiv$
   $\forall x[F] \equiv F$

   (d) (2 points)
   $\neg(\forall z[(P(z) \wedge \neg M(z)) \rightarrow \neg P(z)]) \equiv$
   $\exists z[\neg((P(z) \wedge \neg M(z)) \rightarrow \neg P(z))] \equiv$
   $\exists z[(P(z) \wedge \neg M(z)) \wedge P(z)] \equiv$
   $\exists z[(P(z) \wedge \neg M(z))]$