CS 173: Discrete Structures, Fall 2009
Homework 2

This homework contains 6 problems worth a total of 50 points. It is due on Friday, September 11th at noon. For this week, homeworks can be turned in at class or, after class, at Margaret's office (3214 Siebel). If the door is shut, push the homework under the door. This is a temporary arrangement until they work out the last few details so we can use the newly installed dropboxes.

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1. [**3 points**] **Translating notation into English**

   Suppose we define:

   - $C(x)$ is "x grew up in Chicago."
   - $D(x)$ is "x drives well."
   - $M(x)$ is "x is taking CS 173."
   - $K(x)$ is "x is taking CS 225."
   - S is the set of all students.

   Translate the following into English:

   $$\forall y \in S, (M(y) \wedge \neg K(y)) \rightarrow (D(y) \vee \neg C(y))$$

2. [**9 points**] **Proofs with propositional equivalences**

   Prove that the following pairs of expressions are logically equivalent, using the known equivalences given in Table 6 of Rosen, plus the fact that $p \rightarrow q$ is logically equivalent to $\neg p \vee q$. (Table 6 is reproduced on the handout for lecture 4.)

   (a) $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$

   (b) $\neg p \rightarrow (q \rightarrow \neg r)$ and $r \rightarrow (q \rightarrow p)$

   (c) $(\neg p \vee q) \wedge (r \vee \neg q) \wedge (p \vee q)$ and $q \wedge r$

   Be picky about how you apply the known equivalences, e.g. use an explicit step to simplify double negations, notice that the distributive laws as written in the table require the item being distributed to come before the complex item its being distributed over. However, if you need to rearrange your expression using a sequence of applications of the commutative and associative laws, you don't have to show all the intermediate steps. Just do the rearrangement in one step justified by something like "commutative/associative laws."

3. **[8 points] Negating things**

   Give the negation of the following logical expressions, using logical equivalences to move the "not" operators onto the smallest elements possible. For example, to negate $\forall x, [P(x) \rightarrow Q(x)]$, we first negate the whole thing $\neg\forall x[P(x) \rightarrow Q(x)]$, then convert this to $\exists x[\neg(P(x) \rightarrow Q(x))]$, and finally to $\exists x[P(x) \wedge \neg Q(x)]$. (For simplicity, we've omitted the domains for the quantified variables.)

   (a) $\forall x[\neg M(x) \rightarrow Q(x)]$

   (b) $\exists y[P(y) \wedge (\neg M(y) \vee R(y))]$

   (c) $\exists x[\neg(P(x) \wedge Q(x)) \vee (F(x) \rightarrow P(x)]$

   (d) $\forall z[(P(z) \wedge \neg M(z)) \rightarrow \neg P(z)]$

4. **[10 points] Direct proof and disproof**

   (a) Prove the following claim:

   Claim: For any integer k, if k is odd, then k^3 is odd.

   (b) Disprove the following claim, by giving a concrete counter-example. (You must use a specific counter-example. Don't try to write a general or abstract story about why this equation can't possibly hold.)

   Claim: For any integers p and q, $(p+q)^2 = p^2 + q^2$.

5. **[10 points] Another direct proof**

   Suppose that (x, y) and (p, q) are two intervals of the real line. Let's define (x, y) to *contain* (p, q) if and only if $x \leq p$ and $q \leq y$. Using this definition, prove the following claim:

   Claim: For any intervals of the real line (a, b), (c, d), and (e, f), if (a, b) contains (c, d) and (c, d) contains (e, f), then (a, b) contains (e, f).

6. **[10 points] Son of direct proof**

   (a) Prove the following claim:

   Claim 1: For any integer k, if $k > 4$ then $2k + 1 < k^2$.

   (b) Using your result from part (a), prove the following claim:

   Claim 2: For any integer k, if $k > 4$ and $k^2 < 2^k$, then $(k+1)^2 < 2^{k+1}$.

   Be sure to show the intermediate steps of your algebra carefully, working from the facts you are given towards the facts you are trying to prove. However, you don't have to write justifications for steps that are easy high-school algebra (i.e. most steps in these examples).