

CS 173: Discrete Structures, Fall 2009

Homework 10

This homework contains 4 problems worth a total of 34 points. It is due on Friday, November 20th at noon. Put your homework in the appropriate dropbox in the Siebel basement.

1. Pigeonhole principle [6 points]

A software engineering at Snazzy Mobiles (a startup company) slept for 61 hours over 10 nights. Show that on some pair of consecutive nights, he slept at least 13 hours.

2. Pigeonhole II [10 points]

Over a 30-day period, Frank learns one or more fiddle tunes each day. (Always an integer number of tunes on any given day.) Suppose that he learns no more than 45 tunes over the 30 days. Prove that there is a period of 1 or more consecutive days in which he learns exactly 14 tunes.

Hint: Suppose a_i is the number of tunes learned in all the days up through and including i . Consider the integers a_1, a_2, \dots, a_{30} , together with the integers $a_1 + 14, a_2 + 14, \dots, a_{30} + 14$.

3. Bernoulli Trials [8 points]

Marjorie is still a beginner at flute, so if she tries to play a high A, there is only a 0.3 probability that it will sound. If she makes 5 attempts to play a high A, then...

- What is the chance that it will sound exactly three times?
- What is the chance that it will sound at least three times?
- (4 points) What is the chance that it will sound three times in a row? Include cases in which it sounds more than three times, as long as at least three successes were consecutive. Briefly explain how you calculated your answer and why it's right. Hint: don't blindly apply the Bernoulli Trials formula. Reason from first principles.

4. Expected value [10 points]

Instead of just giving out chocolate on Halloween in the obvious way, your neighbor Mr. Egbert makes you draw cards to see how much he will give you. Mr. Egbert has a deck of 13 cards, with face values 1, 2, ..., 13 and you get to choose a hand containing four cards.

For each hand h , you get $C(h)$ small chocolates, where $C(h)$ is the maximum face value of any of the cards in the hand. For example, you get 9 chocolates if you draw a hand containing 2, 5, 6, and 9.

- How big is our sample space (aka how many possible hands)?
- How many possible hands have k as the maximum value of the cards in the hand? (I.e. give a formula that depends on k .)
- What is the chance of drawing a hand with maximum value k ?
- How many chocolates can you expect to get out of this guy? (I.e. what is the expected value of C ?)