CS 173: Discrete Structures, Spring 2009
Quiz 2 (Wednesday 18 March)

NAME:

NETID:

DISCUSSION DAY/TIME:

This quiz has 3 pages containing 5 questions, totalling 25 points. You have 20 minutes to finish. Showing your work may increase partial credit in case of mistakes.

1. (7 points) Mark the following claims as "true" or "false"

   (a) $n^2 + \log_{10} n = \Theta(n^2)$

   (b) $n = \Theta(n^3)$

   (c) If a function $f(n) = O(g(n))$ then the function $g(n) = \Omega(f(n))$

   (d) If a function $f(n) = O(h(n))$ and a function $g(n) = O(h(n))$ then the function $f(n)g(n) = O(h(n))$

   (e) Let $f : \mathbb{R} \to \mathbb{Z}$ such that $f(x) = \lfloor x \rfloor$. f is onto.

   (f) A function from $\mathbb{N}^2$ to $\mathbb{N}$ cannot be one-to-one because $\mathbb{N}^2$ has more elements than $\mathbb{N}$. (Remember that $\mathbb{N}^2$ is the set of all pairs of natural numbers.)

   (g) Suppose that A and B are sets. If I prove that every element of A is also an element of B, I can conclude that $A = B$.

2. (4 points)

   One of these two statements is true and one is false. State which one is false and explain clearly why it is false.

   (a) For every integer x, there is an integer y such that $y > 3x - 14$.

   (b) There is an integer x, such that for every integer y, $y > 3x - 14$.

3. (4 points) Define the function $f : \mathbb{Z}^2 \to \mathbb{R}$ by $f(x, y) = x + y$. Show that f is not one-to-one by giving a specific counter-example.

4. (4 points) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a function whose inputs and outputs are real numbers. Define what it means for f to be strictly increasing.

5. (6 points)

Let's define a function $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ as follows:

- Base cases: $f(1) = 5$ and $f(2) = 10$
- Induction: $f(n) = 2f(n-1) + f(n-2)$ for all $n \geq 3$

Supply the missing (boxed) parts of the following proof by induction that $f(n) < 3^n$ for all integers $n \geq 3$.

Proof: By induction on n.

Base case or cases:

Inductive hypothesis: [Spell out the specifics of the hypothesis for the inductive step. Don't just refer to "the claim."]

We need to show that our claim holds for $k+1$. We can assume that $(k+1) \geq 5$, since smaller values were covered by the base case(s).

By the definition of f, we know that $f(k+1) = 2f(k) + f(k-1)$.

Applying the induction hypothesis twice, we find that

$$2f(k) + f(k-1) < 2 \cdot 3^k + f(k-1) < 2 \cdot 3^k + 3^{k-1}$$

But $2 \cdot 3^k + 3^{k-1} < 2 \cdot 3^k + 3^k = 3 \cdot 3^k = 3^{k+1}$ by high-school algebra.

Combining these inequalities, we find that $f(k+1) < 3^{k+1}$, which is what we needed to show. $\square$