CS 173, Spring 2009
Midterm 1, 25 February 2009

INSTRUCTIONS (read carefully)

- Fill in your name, netid, and discussion section time below. Also write your netid on the other pages (in case they get separated).

NAME:

NETID:

DISCUSSION DAY/TIME:

- There are 6 problems, on pages numbered 1 through 6. Make sure you have a complete exam. The point value of each problem is indicated next to the problem, and in the table below.
- You have 50 minutes to finish the exam.
- Points may be deducted for solutions which are correct but excessively complicated, hard to understand, or poorly explained.
- Turn in your exam at the front. Show your ID to the proctors.
- It is wise to skim all problems and point values first, to best plan your time.
- This is a closed book exam. No notes or electronic devices of any kind are allowed. Do all work in the space provided, using the backs of sheets if necessary. See the proctor if you need more paper.
- Please bring any apparent bugs to the attention of the proctors.
- After the midterm is over, discuss its contents with other students **only** after verifying that they have also taken the exam (e.g. they aren't about to take a makeup exam).

Problem	1	2	3	4	5	6	total
Possible	12	10	6	8	6	8	50
Score							

Problem 1: Short answer (12 points)

The following require only short answers. Justification/work is not required, but may increase partial credit if your short answer is wrong.

(a) If an if/then statement P is true, is the contrapositive of P always true?

(b) Let $f : \mathbb{R} \rightarrow \mathbb{Z}$ be defined by $f(x) = \lfloor x^2 \rfloor$. Name (or clearly describe) the set that is the co-domain of f.

(c) What is the base-10 equivalent of the hexidecimal number 2F?

(d) Give a closed form expression for $\sum_{k=1}^{p} \frac{1}{2^k}$.

(e) If x is a real number and $\lfloor x \rfloor = \lceil x \rceil$ then what special property must x have?

(f) Suppose A is a set. Is it always the case that the empty set is a member of A?

Problem 2: Set theory (10 points)

(a) (2 points) How many elements are in the set $\{\{3\}, \{4, 5\}, \emptyset\}$?

Compute the output of each of the following set operations. Recall that $\mathbb{P}(A)$ is the power set of A.

(b) (3 points) $\mathbb{P}(\{a, b, c\}) =$

(c) (3 points) $\{4, 7\} \times \{7, 9, 2\} =$

(d) (2 points) $\{2, 8, 4\} \times \emptyset =$

Problem 3: Longer answers (6 points)

(a) Trace the execution of the Euclidean algorithm as it computes the GCD of 245 and 280.

(b) Suppose that A and B are sets. Prove that $\mathbb{P}(A \cup B)$ is not always equal to $\mathbb{P}(A) \cup \mathbb{P}(B)$ by giving a specific counter-example and explaining briefly why it is a counter-example.

Problem 4: Remembering definitions (8 points)

(a) If d is a positive integer, the quotient and remainder modulo d are defined by a theorem named the "division algorithm." Finish the following statement of this theorem:

> Suppose that a is an integer and d is a positive integer, then there are unique integers q and r such that ...

(b) Suppose that m and n are integers, not both zero. Define what it means for an integer d to be the GCD of m and n. Assume that we already know what $a|b$ means and base your answer on this "divides" relation. Do not use prime factorizations.

Problem 5: Writing a proof (6 points)

Recall the following definition: Given any positive integer m, the integers a and b are *congruent modulo* m if and only if there is an integer k such that $a = b + km$. a is congruent to b modulo m is written as $a \equiv b \pmod{m}$.

Prove that, for any integers p, q, and r and any positive integer m,

If $p \equiv q \pmod{m}$ and $q \equiv r \pmod{m}$, then $p \equiv r \pmod{m}$.

Prove this directly using the above definition, together with high school algebra. Do not use other facts about modular arithmetic proved in class or in the book.

Problem 6: Logic and proof structure (8 points)

1. (3 points) Give the negation of the following statement, moving the negatives (e.g. "not") so that they are on individual predicates (e.g. "my bike has two wheels"). Although you can use shorthand to work out your answer, your final answer must be written out in words. Everything after the word "then" is intended to be read as belonging to the conclusion of the if/then statement.

   For every book B, if B is a fantasy novel, then B must involve magic and B must feature a naive young hero.

2. (5 points) Prove the following claim using proof by contradiction. You **must** use proof by contradiction.

   For any natural numbers p and q, if $pq - 17 < 80$ then either $p < 10$ or $q < 10$.