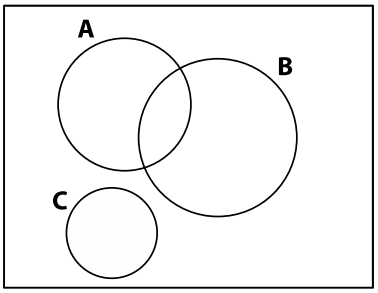
Homework #1

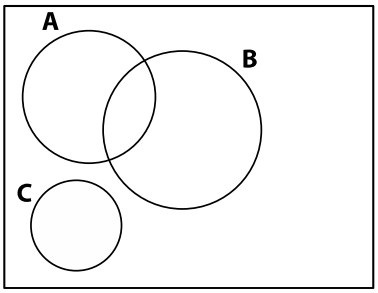
1. (**10 points**) Events A and B are mutually exclusive and have probabilities $P(A) = 0.2$, $P(B) = 0.2$. Are they independent?

2. (**10 points**) Three events are shown on the Venn diagram in the following figure:

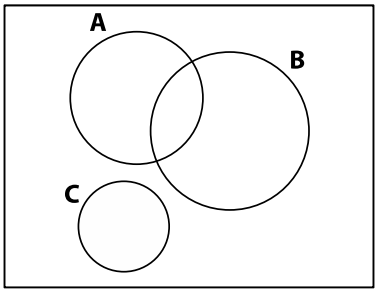


For each of the questions (a-e) reproduce the figure and shade the region corresponds to the following events.
(a) A' (b) $(A \cap B) \cup (A \cap B')$ (c) $(A \cap B) \cup C$ (d) $(B \cup C)'$ (e) $(A \cap B)' \cup C$

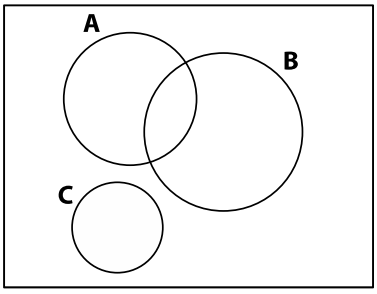
(a)



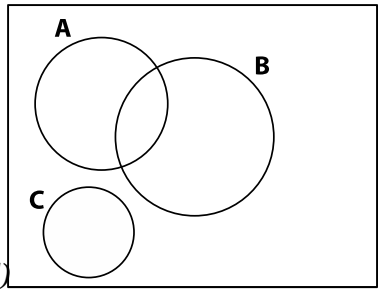
(b)



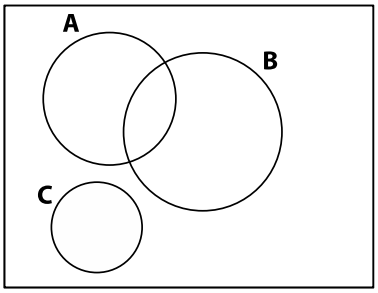
(c)



(d)



(e)



3. **(5 points)** The hospital emergency department data are summarized in the following table. Let A

denote the event that a visit was to the Hospital 1 and let B denote the event that a visit resulted in admittance to any hospital. Determine the number of people in each of the following events. (a) $A \cap B$ (b) A' c) $A \cup B$ (d) $A \cup B'$ (e) $A' \cap B'$

Hospital	1	2	3	4	total
Total	5292	6991	5640	4329	22,252
Left before seen by a doctor	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

4. **(10 points)** There are 4 red balls and 6 white balls in a box. One draws two balls simultaneously. What is the probability that they are of the same color?

5. **(20 points)** George asked his professor for a recommendation letter for graduate school. He estimates that the probability that the letter will be strong is 0.5, the probability that the letter will be weak is 0.2, and mediocre is 0.3. He also estimates that if the letter is strong, the probability that he will be accepted to graduate school is 0.8; if it is weak – it is exactly 0; and if it is mediocre, the probability is 0.4. Given that he did get accepted to the only school he applied, find the probability that: (a) the letter was strong and (b) the letter was weak?

6. **(10 points)** Suppose that a bag contains ten coins, three of which are fair, while the remaining seven are biased: they have probability of 0.6 of heads when flipped. A coin was taken at random from the bag and flipped five times. All five flips gave heads. What's the probability that this coin is fair?

7. (**5 points**) The following circuit operates if and only if there is at least one path of functional devices from left to right. The probability that each device is functional is as shown in each box. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?