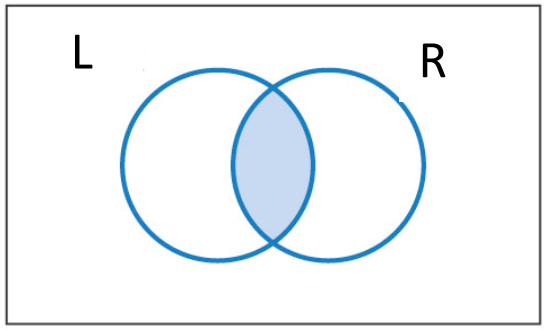
Circuits

Series Circuit

This circuit operates only if there is at least one path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that the devices fail independently. What is the probability that the circuit operates?

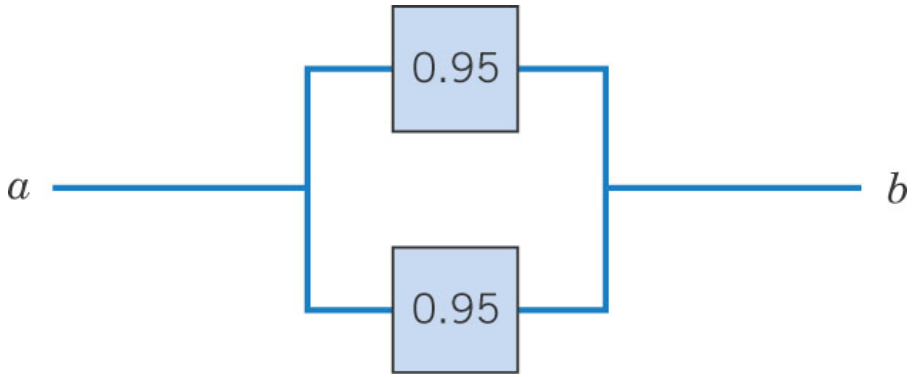


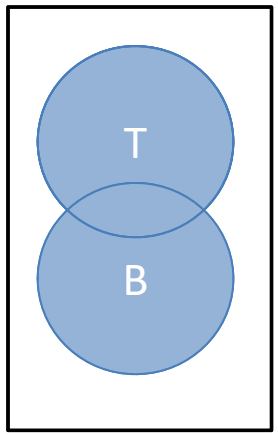
Let L & R denote the events that the left and right devices operate. The probability that the circuit operates is:

$P(L \text{ and } R) = P(L \cap R) = P(L) * P(R) = 0.8 * 0.9 = 0.72.$

Parallel Circuit

This circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown. Each device fails independently.



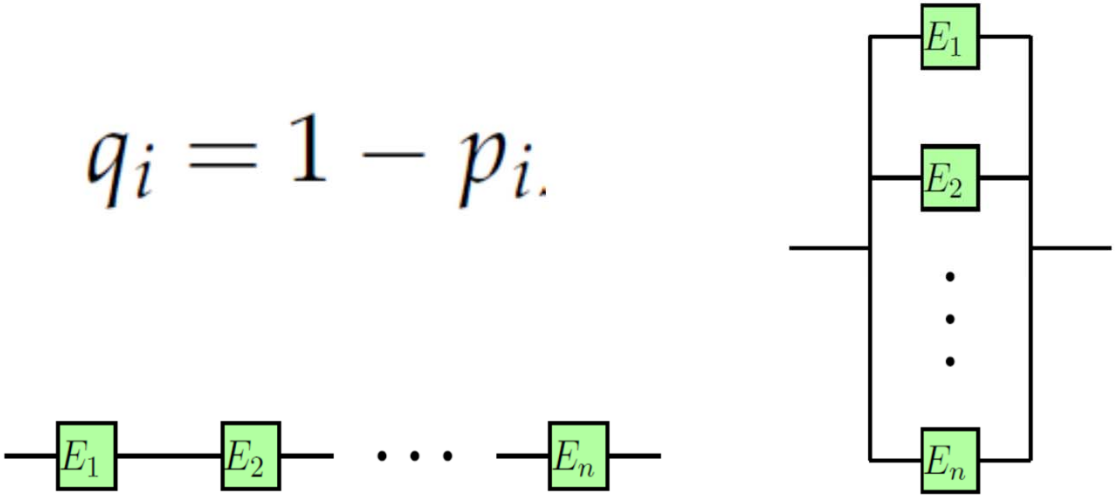


Let T & B denote the events that the top and bottom devices operate. The probability that the circuit operates is:

$P(T \cup B) = 1 - P(T' \cap B') = 1 - P(T')*P(B') = 1 - 0.05^2 = 1 - 0.0025 - 0.9975.$

Duality between parallel and series circuits

$$q_i = 1 - p_i.$$

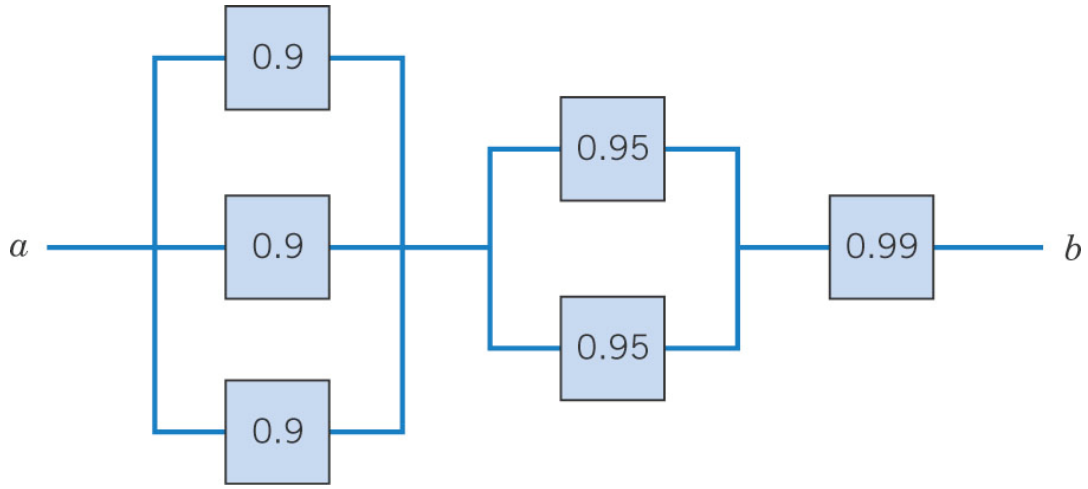


(a) (b)

Connection	Notation	Works with prob	Fails with prob
Serial	$E_1 \cap E_2 \cap \cdots \cap E_n$	$p_1 p_2 \ldots p_n$	$1 - p_1 p_2 \ldots p_n$
Parallel	$E_1 \cup E_2 \cup \cdots \cup E_n$	$1 - q_1 q_2 \ldots q_n$	$q_1 q_2 \ldots q_n$

Advanced Circuit

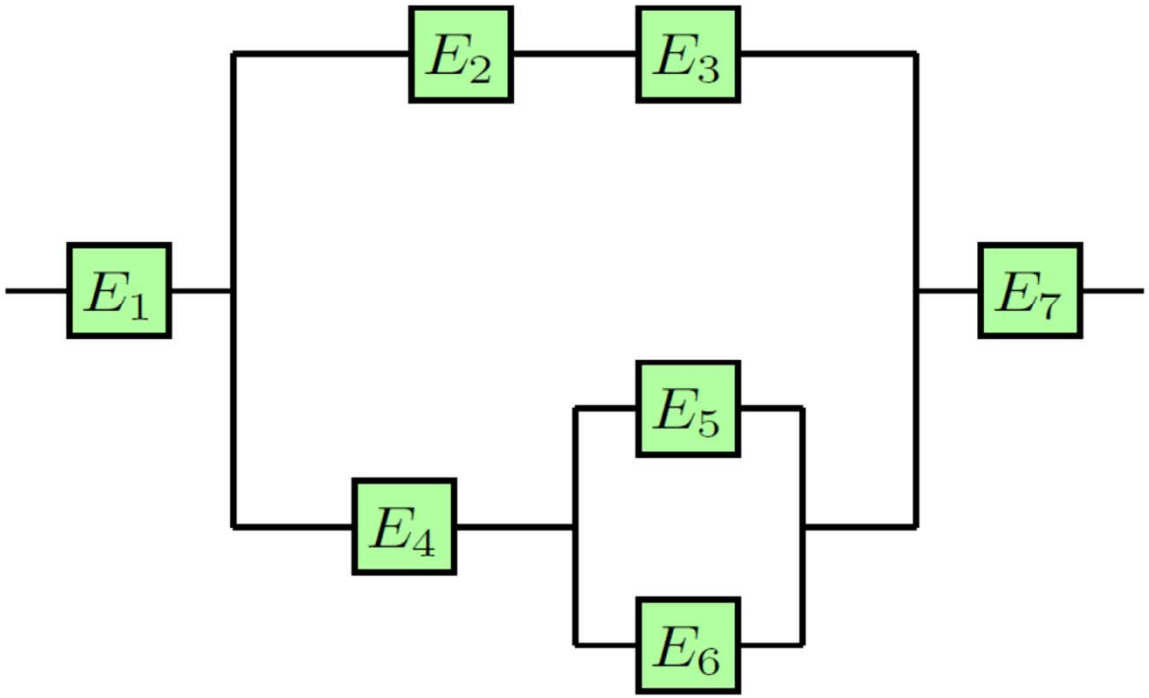
This circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown. Each device fails independently.



Partition the graph into 3 columns with L & M denoting the left & middle columns.

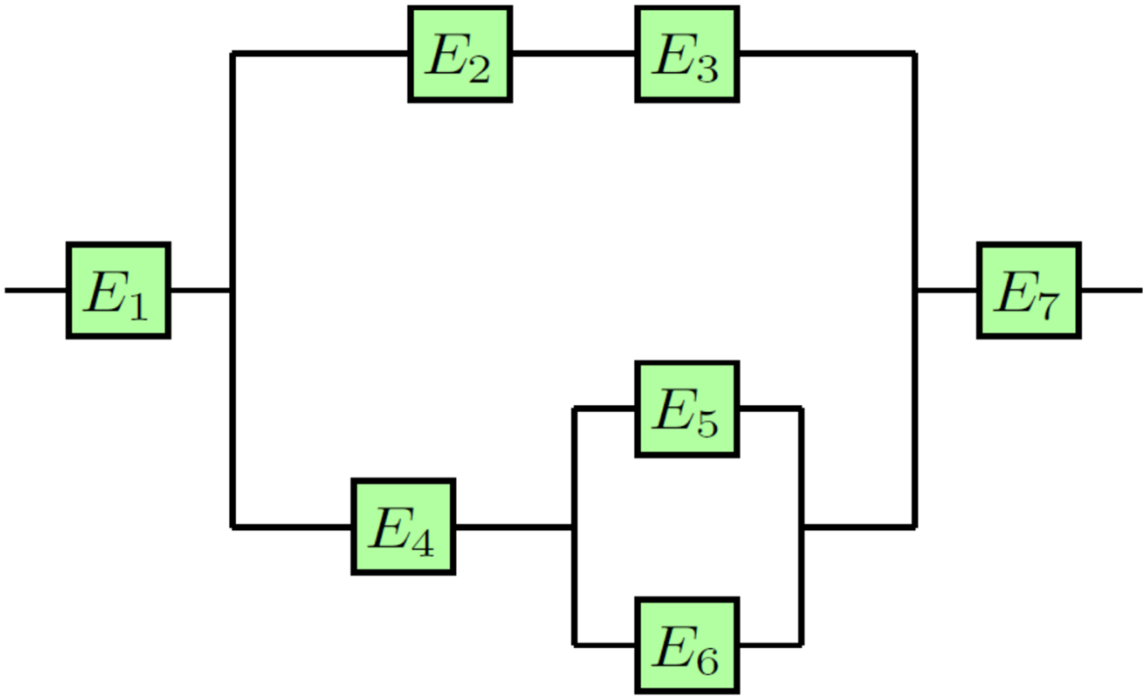
P(L) = 1- 0.1^3 , and P(M) = 1- 0.05^2, so the probability that the circuit operates is: $(1 - 0.1^3)(1-0.05^2)(0.99) = 0.9875$ (this is a series of parallel circuits).

Circuit → Set equation



Component	E_1	E_2	E_3	E_4	E_5	E_6	E_7
Probability of functioning well	0.9	0.5	0.3	0.1	0.4	0.5	0.8

Circuit → Set equation



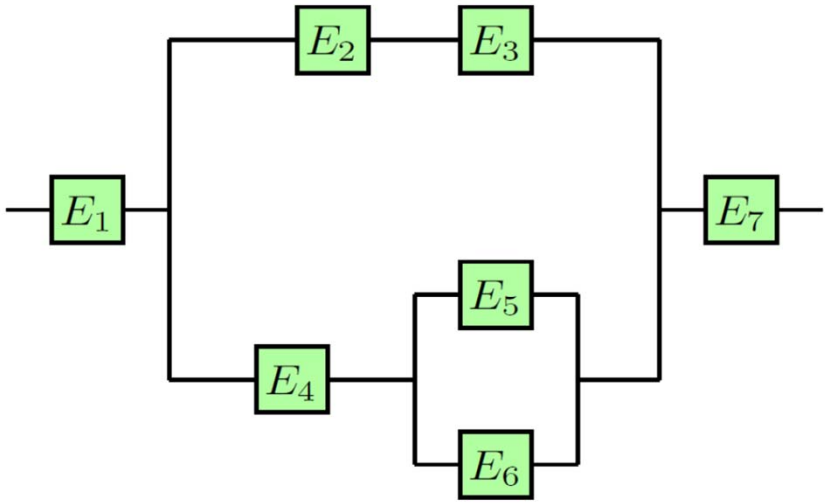
Component	E_1	E_2	E_3	E_4	E_5	E_6	E_7
Probability of functioning well	0.9	0.5	0.3	0.1	0.4	0.5	0.8

$$E_1 \cap [(E_2 \cap E_3) \cup (E_4 \cap (E_5 \cup E_6))] \cap E_7.$$

P(Works) = 0.9.*(1-(1-0.5.*0.3).*(1-0.1.*(1-0.6.*0.5))).*0.8=0.15084

Matlab group exercise

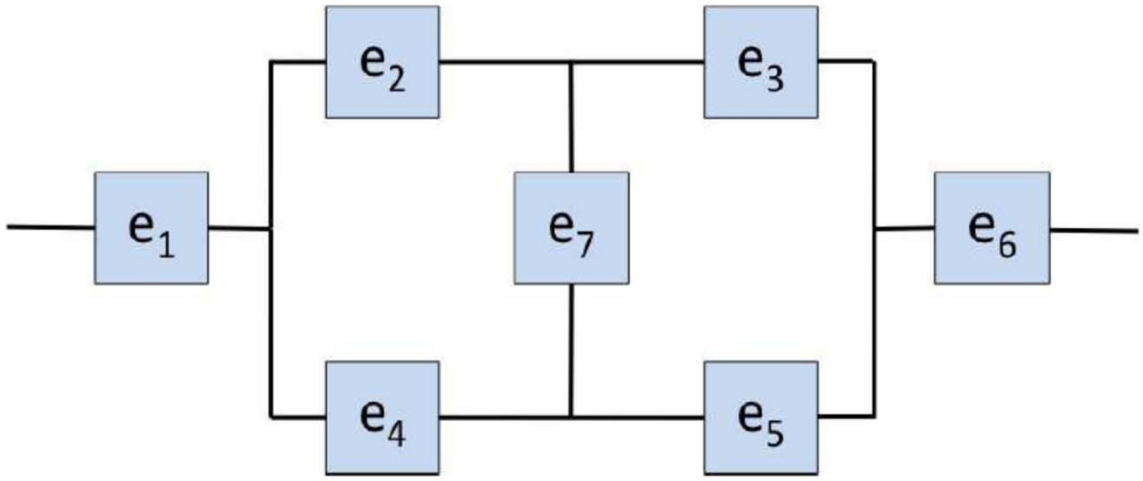
- Break into groups by rows (or half rows).
  I will collect results on the blackboard
- Each group writes a matlab script to simulate 10^6 tests of this circuit
- Calculate the fraction of time circuit works
- Compare to our calculation: 0.15084



Component	E_1	E_2	E_3	E_4	E_5	E_6	E_7
Probability of functioning well	0.9	0.5	0.3	0.1	0.4	0.5	0.8

Matlab check

- M=1000000;
- s = 0;
- for i = 1:M
- e1 = rand < 0.9; e2 = rand < 0.5; e3 = rand < 0.3;
- e4 = rand < 0.1; e5 = rand < 0.4; e6 = rand < 0.5;
- e7 = rand < 0.8;
- s1 = min(e2,e3); % or s1 = e2*e3;
- s2 = max(e5,e6); % or s2= e5+e6>0;
- s3 = min(e4,s2); % or s3 = e4*s2;
- s4 = max(s1,s3); % or s4 = s1+s3 > 0;
- st = min([e1;s4;e7]); % or st=e1*s4*e7;
- s = s + st;
- end
- works = s/M
- fails = 1 - works



Component	e_1	e_2	e_3	e_4	e_5	e_6	e_7
Probability of component working	0.3	0.8	0.2	0.2	0.5	0.6	0.4

Bayes Theorem

Bayes' theorem (1812)

Thomas Bayes
(1701-1761)
English statistician, philosopher,
and Presbyterian minister

Bayes' theorem (simple)

Definitions: P(B|A) =P(B∩A)/P(A); P(A|B) =P(A∩ B)/P(B)

$$P(A \cap B) = P(A|B)P(B) = P(B \cap A) = P(B|A)P(A)$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- In Science we often want to know:
  "How much faith should I put into hypothesis, given the data?"
  or $P(H|D)$
- What we usually can calculate is:
  "Assuming that this hypothesis is true, what is the probability of the observed data?" or $P(D|H)$
- Bayes' theorem can help: $P(H|D)=P(D|H)\bullet P(H)/P(D)$
- The problem is $P(H)$ (so-called prior) is often not known

Bayes theorem (continued)

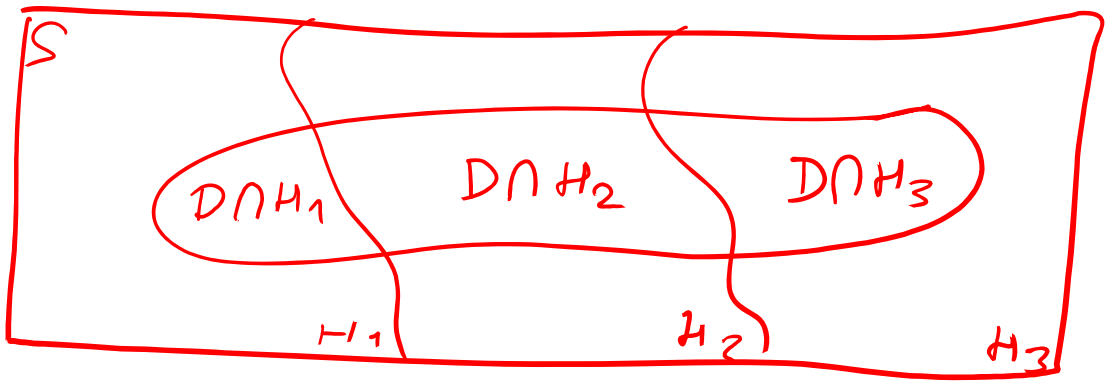
Works best with exhaustive and mutually-exclusive hypotheses:
$H_1, H_2, \ldots H_n$ such that $H_1 \cup H_2 \cup H_3 \ldots \cup H_n = S$ and $H_i \cap H_j = \varnothing$ for $i \neq j$

$P(H_k|D) = P(D|H_k) \bullet P(H_k)/P(D)$

- $P(H_k)$ is a prior of hypothesis k. But what is P(D)?

- $P(D) = P(D \cap H_1) + P(D \cap H_2) + \ldots P(D \cap H_n) =$

$= P(D|H_1) \bullet P(H_1) + P(D|H_2) \bullet P(H_2) + \ldots P(D|H_n) \bullet P(H_n)$



An **awesome new test** has been invented for an early detection of cancer. The probability that it correctly identifies someone with cancer as positive is 95%, and the probability that it correctly identifies someone without cancer as negative is 99%. The incidence of this type of cancer in the general population is 10^{-4}. A random person in the population takes the test, and the result is positive.

What is the probability that he/she has cancer?

A. 99%
B. 95%
C. 30%
D. 1%

Get your i-clickers

Events: C - cancer, C' - no cancer

Test events Y - positive, N - negative

We know:

$$P(C) = 10^{-4}, \quad P(Y|C) = 0.95$$
$$P(N|C') = 0.99$$

We need

$$P(C|Y)$$

Bayes:

$$P(C|Y) = P(Y|C) \cdot \frac{P(C)}{P(Y)} \; ?$$

$P(Y)$ – probability that a random person will test positive

$$P(Y) = P(Y \cap C) + P(Y \cap C') =$$

$$= P(Y|C)P(C) + P(Y|C')P(C') =$$

$$= 0.95 \times 10^{-4} + (1 - 0.99) \times (1 - 10^{-4}) \approx$$

$$\approx 10^{-4} + 10^{-2} \approx 10^{-2} = 1\%$$

$$P(C|Y) = P(Y|C) \cdot \frac{P(C)}{P(Y)} = 0.95 \times \frac{10^{-4}}{10^{-2}} \approx$$

$$\approx 1\%$$

What if a doctor is already 50% sure of cancer based on other tests?

That changes things!

Now $P(C) = P(C') = 0.5$

$$P(C|Y) = \frac{P(Y|C) \cdot P(C)}{P(Y|C) \cdot P(C) + P(Y|C') P(C')} =$$

$$= \frac{0.95 \times 0.5}{0.95 \times 0.5 + (1-0.99) \times 0.5} \approx 0.99$$

How come?

We thought it was a great test..

- Let C – be the event that the patient has cancer; C’ – patient is cancer free
- Y/N – events that test is Positive/Negative (N=Y’)
- We know: $P(C)=10^{-4}$, $P(Y|C)=0.95$, $P(N|C')=0.99$
- We need to find $P(C|Y)$
- Bayes to the rescue: $P(C|Y)=P(Y|C)*P(C)/P(Y)$
- What on earth is P(Y) ???

What is P(Y) ???

- Likelihood that a random patient would test Y:
  $P(Y)=P(Y \cap C)+P(Y \cap C')=P(Y|C)P(C)+P(Y|C')P(C')=0.95*10^{-4}+(1-0.99)*(1-10^{-4})\approx 0.01$
- Hence $P(C|Y)=P(Y|C)*P(C)/P(Y) \approx 10^{-4}/0.01=0.01=1\%$
- But we would like it to be 100%, please!!!
- Until the false positive discovery rate $1-P(N|C')$ does not fall below the general population prevalence the result will never be close 100%

What if I am already 50% sure (based on other tests) that a patient has cancer?

- That changes everything!
- Now P(C)=P(C')=0.5
- P(C|Y)=P(Y|C)*P(C)/[P(Y|C)*P(C)+ P(Y|C')*P(C')]=

0.95*0.5/[0.95*0.5+(1-0.99)*0.5]=0.99

- Now the doctor can be almost 100% sure.
- The importance of prior:
  - If prior belief that one has cancer is 10^{-4} – test is useless
  - If prior belief is at least 1% - the test is useful

(15 points) Prostate cancer is the most common type of cancer found in males. It is checked by PSA test that is notoriously unreliable. The probability that a noncancerous man will have an elevated PSA level is approximately 0.135, with this probability increasing to approximately 0.268 if the man does have cancer. If, based on other factors, a physician is 70 percent certain that a male has prostate cancer, what is the conditional probability that he has the cancer given that the test indicates an elevated PSA level?

A. 99.99%

B. 99%

C. 82%

D. 71%

Get your i-clickers

Use Bayes Theorem again

Events: C – cancer, E – PSA elevated

$P(C) = 0.7$: doctor's prior belief

$$P(C|E) = P(E|C) \cdot \frac{P(C)}{P(E)}$$

$$P(E) = P(E|C) \cdot P(C) + P(E|C') \cdot P(C')$$

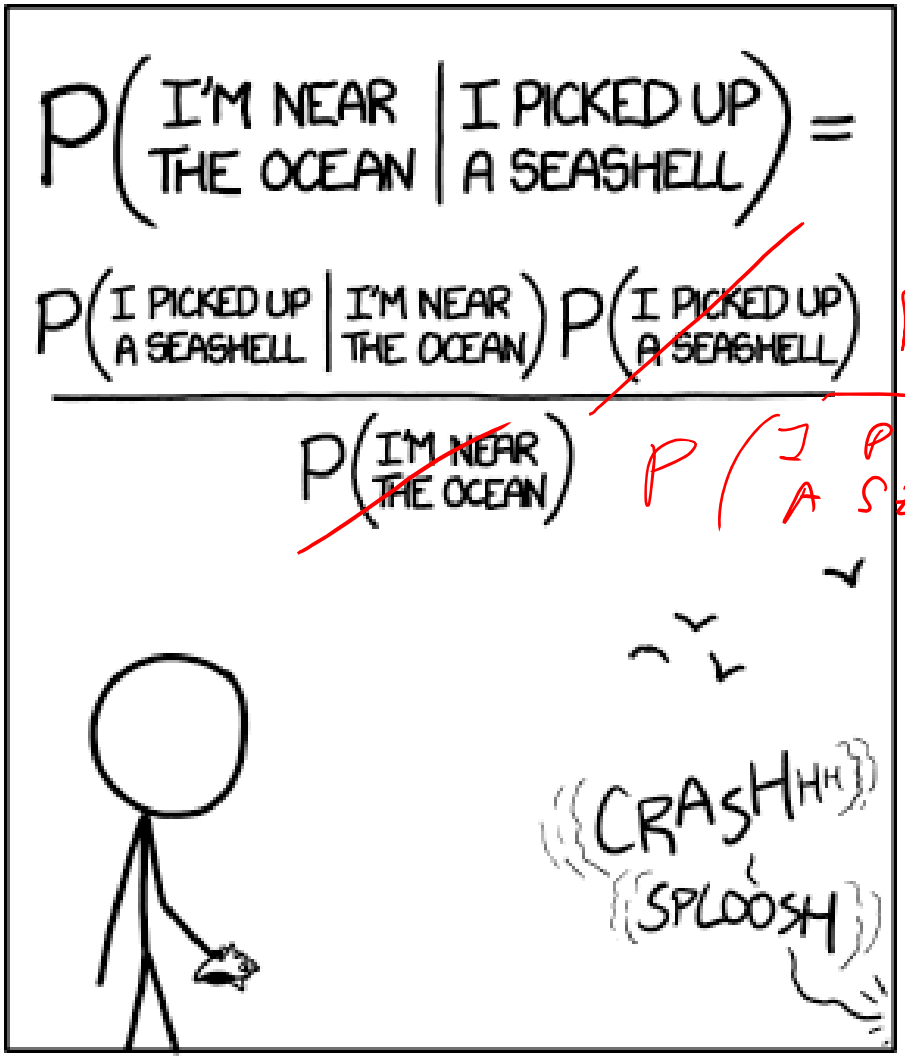
$$= 0.268 \cdot 0.7 + 0.135 \cdot 0.3 = 0.23$$

$$P(C|E) = 0.268 \cdot \frac{0.7}{0.23} = 0.82 = \underline{82\%}$$

vs. 70%

All this trouble for a lousy 12% gain in confidence? I don't believe you!

- Let C – be the event that the patient has cancer; C' – patient is cancer free, E – events that the PSA test was elevated
- We know **doctor's prior belief**: P(C)=0.7
- We know test stats: P(E|C)=0.268, P(E|C')=0.135
- We need to find P(C|E)=P(E|C)*P(C)/P(E)
- P(E)=P(E|C)*P(C)+P(E|C')*P(C')= =0.268*0.7+0.135*0.3=0.23
- P(C|E)=0.7*0.268/0.23=0.82=82%

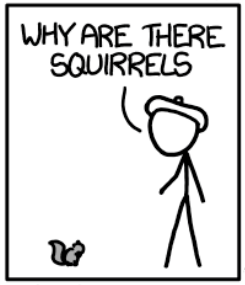


What is wrong in this comics?

If you are not yet reading XKCD comics
https://xkcd.com/
you should start

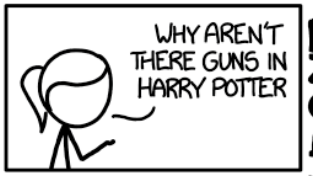
Credit: XKCD comics

QUESTIONS
FOUND IN GOOGLE AUTOCOMPLETE

WHY DO WHALES JUMP
WHY ARE WITCHES GREEN
WHY ARE THERE MIRRORS ABOVE BEDS
WHY DO I SAY UH
WHY IS SEA SALT BETTER
WHY ARE THERE TREES IN THE MIDDLE OF FIELDS
WHY IS THERE NOT A POKEMON MMO
WHY IS THERE LAUGHING IN TV SHOWS
WHY ARE THERE DOORS ON THE FREEWAY
WHY ARE THERE SO MANY SVCHOST.EXE RUNNING
WHY AREN'T THERE ANY COUNTRIES IN ANTARCTICA
WHY ARE THERE SCARY SOUNDS IN MINECRAFT
WHY IS THERE KICKING IN MY STOMACH
WHY ARE THERE TWO SLASHES AFTER HTTP
WHY ARE THERE CELEBRITIES
WHY DO SNAKES EXIST
WHY DO OYSTERS HAVE PEARLS
WHY ARE DUCKS CALLED DUCKS
WHY DO THEY CALL IT THE CLAP
WHY ARE KYLE AND CARTMAN FRIENDS
WHY IS THERE AN ARROW ON AANG'S HEAD
WHY ARE TEXT MESSAGES BLUE
WHY ARE THERE MUSTACHES ON CLOTHES
WHY ARE THERE MUSTACHES ON CARS
WHY ARE THERE MUSTACHES EVERYWHERE
WHY ARE THERE SO MANY BIRDS IN OHIO
WHY IS THERE SO MUCH RAIN IN OHIO
WHY IS OHIO WEATHER SO WEIRD
WHY ARE THERE MALE AND FEMALE BIKES
WHY ARE THERE BRIDESMAIDS
WHY DO DYING PEOPLE REACH UP
WHY AREN'T THERE VARICOSE ARTERIES
WHY ARE OLD KLINGONS DIFFERENT
WHY ARE THERE SQUIRRELS

WHY IS PROGRAMMING SO HARD
WHY IS THERE A 0 OHM RESISTOR
WHY DO AMERICANS HATE SOCCER
WHY DO RHYMES SOUND GOOD
WHY DO TREES DIE
WHY IS THERE NO SOUND ON CNN
WHY AREN'T POKEMON REAL
WHY AREN'T BULLETS SHARP
WHY DO DREAMS SEEM SO REAL
WHY ARE THERE TINY SPIDERS IN MY HOUSE
WHY DO SPIDERS COME INSIDE
WHY ARE THERE HUGE SPIDERS IN MY HOUSE
WHY ARE THERE LOTS OF SPIDERS IN MY HOUSE
WHY ARE THERE SPIDERS IN MY ROOM
WHY ARE THERE SO MANY SPIDERS IN MY ROOM
WHY DO SPIDER BITES ITCH
WHY IS DYING SO SCARY
WHY IS THERE NO GPS IN LAPTOPS
WHY DO KNEES CLICK
WHY AREN'T THERE E GRADES
WHY IS ISOLATION BAD
WHY DO BOYS LIKE ME
WHY DON'T BOYS LIKE ME
WHY IS THERE ALWAYS A JAVA UPDATE
WHY ARE THERE RED DOTS ON MY THIGHS
WHY IS LYING GOOD
WHY IS SEX SO IMPORTANT

WHY ARE THERE SLAVES IN THE BIBLE
WHY DO TWINS HAVE DIFFERENT FINGERPRINTS
WHY ARE AMERICANS AFRAID OF DRAGONS
WHY IS HTTPS CROSSED OUT IN RED
WHY IS THERE A LINE THROUGH HTTPS
WHY IS THERE A RED LINE THROUGH HTTPS ON FACEBOOK
WHY IS HTTPS IMPORTANT
WHY AREN'T MY ARMS GROWING

WHY ARE THERE WEEKS
WHY DO I FEEL DIZZY
WHY IS YKK ON ALL ZIPPERS
WHY ARE THERE SWARMS OF GNATS
WHY IS THERE PHLEGM
WHY ARE THERE SO MANY CROWS IN ROCHESTER, MN
WHY AREN'T ECONOMISTS RICH
WHY DO AMERICANS CALL IT SOCCER
WHY ARE MY EARS RINGING
WHY ARE THERE SO MANY AVENGERS
WHY ARE THE AVENGERS FIGHTING THE X MEN
WHY IS WOLVERINE NOT IN THE AVENGERS
WHY IS THERE LAVA
WHY IS PSYCHIC WEAK TO BUG
WHY DO CHILDREN GET CANCER
WHY IS POSEIDON ANGRY WITH ODYSSEUS
WHY IS THERE ICE IN SPACE
WHY ARE THERE ANTS IN MY LAPTOP
WHY IS EARTH TILTED
WHY IS SPACE BLACK
WHY IS OUTER SPACE SO COLD
WHY ARE THERE PYRAMIDS ON THE MOON
WHY IS NASA SHUTTING DOWN
WHY ARE THERE GHOSTS

WHY IS THERE AN OWL IN MY BACKYARD
WHY IS THERE AN OWL OUTSIDE MY WINDOW
WHY IS THERE AN OWL ON THE DOLLAR BILL
WHY DO OWLS ATTACK PEOPLE
WHY ARE AK 47S SO EXPENSIVE
WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE
WHY ARE THERE GODS
WHY ARE THERE TWO SPOCKS
WHY IS MT VESUVIUS THERE
WHY DO THEY SAY T MINUS
WHY ARE THERE OBELISKS
WHY ARE WRESTLERS ALWAYS WET
WHY ARE OCEANS BECOMING MORE ACIDIC
WHY IS ARWEN DYING
WHY AREN'T MY QUAIL LAYING EGGS
WHY AREN'T MY QUAIL EGGS HATCHING
WHY AREN'T THERE ANY FOREIGN MILITARY BASES IN AMERICA
WHY ARE CIGARETTES LEGAL
WHY ARE THERE DUCKS IN MY POOL
WHY IS JESUS WHITE
WHY IS THERE LIQUID IN MY EAR
WHY DO Q TIPS FEEL GOOD
WHY DO GOOD PEOPLE DIE
WHY AREN'T THERE GUNS IN HARRY POTTER

WHY ARE ULTRASOUNDS IMPORTANT
WHY ARE ULTRASOUND MACHINES EXPENSIVE
WHY IS STEALING WRONG
WHY ARE DOGS AFRAID OF FIREWORKS
WHY IS THERE NO KING IN ENGLAND
WHY IS LIFE SO BORING
WHY ARE THERE FEMALE MR MIMES
WHY IS GPS FREE
WHY IS THERE HELL IF GOD FORGIVES
WHY DO IGUANAS DIE
WHY AREN'T THERE DINOSAUR GHOSTS