Preparing for the Midterm Exam

- Thursday, November 9, 2pm-3:20pm
- Here in 305 MSE
- Closed book exam; no books, notes, laptops, etc.
- However, calculators can be used.
- You can prepare one cheat sheet (letter size)

This will be provided

Name	Probability Distribution	Mean	Variance	Section in Book
Discrete				
Uniform	$\frac{1}{n}, a \le b$	$\frac{(b+a)}{2}$	$\frac{(b-a+1)^2-1}{12}$	3-5
Binomial	$\binom{n}{x}p^x(1-p)^{n-x},$	np	np(1-p)	3-6
	$x=0,1,\ldots,n,0\leq p\leq 1$			
Geometric	$(1-p)^{x-1}p,$ $x = 1, 2, \dots, 0 \le p \le 1$	1/p	$(1-p)/p^2$	3-7.1
Negative binomial	$\binom{x-1}{r-1}(1-p)^{x-r}p^r$	r/p	$r(1-p)/p^2$	3-7.2
	$x = r, r + 1, r + 2, \dots, 0 \le p \le 1$			
H	$\frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{x}}$		$np(1-p)\binom{N-n}{N-1}$	3-8
Hypergeometrie	(N)	np, K	np(1-p)(N-1)	3-8
	(n)	where $p = \frac{K}{N}$		
	$x = \max(0, n - N + K) $ $\min(K, n), K \le N, n \le N$			
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \dots, 0 < \lambda$	λ	λ	3-9
Continuous				
Uniform	$\frac{1}{b-a}, a \le x \le b$	$\frac{(b+a)}{2}$	$\frac{(b-a)^2}{12}$	4-5
Normal	$\frac{1}{\sigma\sqrt{2\pi}}e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2	4-6
	$-\infty < \chi < \infty, -\infty < \mu < \infty, 0 < \sigma$			
Exponential	$\lambda e^{-\lambda x}, 0 \le x, 0 < \lambda$	$1/\lambda$	$1/\lambda^2$	4-8
Erlang	$\frac{\lambda^r x^{r-1} e^{-\lambda x}}{(r-1)!}, 0 < x, r = 1, 2, \dots$	r/λ	r/λ^2	4-9.1
Gamma	$\frac{\lambda' x^{r-1} e^{-\lambda x}}{\Gamma(r)}, 0 < x, 0 < r, 0 < \lambda$	r/λ	r/λ^2	4-9.2

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0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785230
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.81326
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.88297
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901473
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.93188
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.94408
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.95448
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.96327
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.97062
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.97670
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.98169
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985733
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.98898
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.99520
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.99642
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.99736
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.99807
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998603
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.99949
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.99983
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.99988
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.99992
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.99996

What may be on the exam?

- Bayes Theorem
- Discrete & Continuous Random Variables
- Joint Probability Distributions, Covariance/Correlations
- Confidence Intervals
- Hypothesis testing
- Other topics
- Look at Homeworks 1-4 for examples of problems

Hypothesis testing

3. (8 points) The college bookstore tells prospective students that the average cost of its textbooks is \$52 with a standard deviation of \$4.50. A group of statistics students think that the average cost is <u>actually higher</u>. In order to test bookstore's claim against this alternative hypothesis, the students bought a random sample of 100 books. The mean price of this sample was \$52.80. Perform the hypothesis test at the 5% level of significance and state your decision.

What type of hypothesis should I apply?

- A. Two-sided: $\mu_1 \neq \mu_0$
- B. One-sided: $\mu_1 > \mu_0$
- C. One-sided: $\mu_1 < \mu_0$
- D. Three-sided
- E. I have no idea

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The standard deviation of \bar{x} in this sample is:

- A. \$4.50
- B. \$45
- C. \$0.45
- D. I have no idea

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Answer: Hypothesis: $\begin{cases} H_0: \mu = 52 \\ H_1: \mu > 52 \end{cases}$ The critical z-value can be obtained from $z^* = \frac{52.8 - 52}{4.5/10} = 1.78 \text{ Since } z^* > z_{\alpha} = 1.65 \text{, this test statistic lies in the rejection region for H₀. Thus$

null hypothesis H₀ will be rejected and alternative hypothesis H₁ is accepted.

Confidence intervals

- **1.** A sample of size 100 which has the sample mean $\overline{X} = 500$ was drawn from a population with an unknown mean μ and the standard deviation $\sigma = 80$.
 - a) Give the 95% confidence interval for the population mean.

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 - a) Give the 95% confidence interval for the population mean.

Answer:
$$P(500 - z_{0.025} \frac{80}{\sqrt{100}} < Z < 500 + z_{0.025} \frac{80}{\sqrt{100}}) = 0.95$$
 where $z_{0.025} = 1.96$

Therefore, the interval is [484.32, 515.68].

Bayes rule

In answering a question on a multiple-choice test, a student either knows the answer or he guesses. Let 1/3 be the probability that he knows the answer. If he does not know the answer, he randomly guesses one out of 4 multiple choice questions. What is the conditional probability that a student knew the answer to a question given that he answered it correctly?

- A. 1/4
- B. 1/3
- C. 2/3
- D. 1/5
- E. I don't know

Get your i-clickers

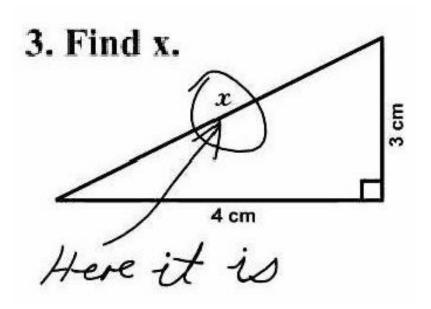
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Answer: P(K)=1/3, P(K')=2/3, P(C|K)=1, P(C|K')=1/4. P(K|C)=P(C|K)*P(K)/P(C)=1*(1/3)/(1*1/3+(1/4)*(2/3))=2/3=0.666...

Discrete Probability Distributions

What is X in this problem?

- What is the random variable: Look for keywords:
 - Find the probability that....
 - What is the mean (or variance) of...
- What are parameters? Look for keywords:
 - Given that...
 - Assuming that...



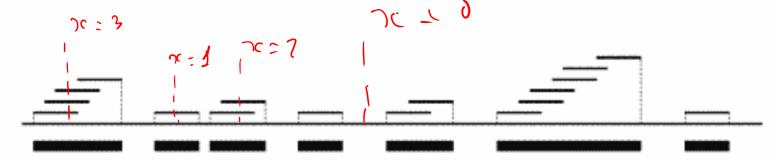
O'cide to probability distributions Bluomial: # of samples, n, is fixed, # of successes, x, is variable $\mathcal{P}(X=x) = \frac{n!}{2c!(n-2c)!} p^{2c}(1-p)^{n-2c}$ · Geometric: # of samples, x is Variable. # of successes 1 is fixed. Success comes in the end $\mathcal{P}(X=x) = (1-p)^{x-1} \cdot p$ Negative binomial: # of samples, x is variable. # of successes, Γ , is fixed rth success in + be end (x-1)! $p'(1-p)^{x-r}$ P(X=x) = (r-1)!(x-r)!

Poisson distribution in genomics

- G genome length (in bp)
- L short read average length
- N number of short read sequenced
- λ sequencing redundancy = LN/G
- x- number of short reads covering a given site on the genome

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$
 Ewens, Grant, Chapter 5.1

Poisson as a limit of Binomial. For a given site on the genome for each short read Prob(site covered): p=L/G is very small. Number of attempts (short reads): N is very large. Their product (sequencing redundancy): $\lambda = NL/G$ is O(1).



Probability that a base pair in the genome is not covered by any short reads is 0.1

One randomly selects base pairs until exactly 5 uncovered base pairs are found.

Which discrete probability distribution describes the number of attempts?

- A. Poisson
- B. Binomial
- C. Geometric
- D. Negative Binomial
- E. I have no idea

Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2,, 0 < \lambda$
Binomial	$\binom{n}{x} p^x (1-p)^{n-x}$
	$x = 0, 1,, n, 0 \le p \le 1$
Geometric	$(1-p)^{x-1}p$ $x = 1, 2,, 0 \le p \le 1$
Negative binomial	$\binom{x-1}{r-1} (1-p)^{x-r} p^r$
	$x = r, r + 1, r + 2,, 0 \le p \le 1$

Probability that a base pair in the genome is not covered by any short reads is 0.1

One randomly selects base pairs until exactly 5 uncovered base pairs are found.

What are the values of p, r?

A.
$$p=0.5, r=5$$

B.
$$p=0.1$$
, $r=0.5$

D.
$$p=0.5$$
, $r=0.1$

Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2,, 0 < \lambda$
Binomial	$\binom{n}{x} p^x (1-p)^{n-x}$
	$x = 0, 1,, n, 0 \le p \le 1$
Geometric	$(1-p)^{x-1}p$ $x = 1, 2,, 0 \le p \le 1$
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Negative binomial	$\binom{x-1}{r-1} (1-p)^{x-r} p^r$
	$x = r, r + 1, r + 2,, 0 \le p \le 1$

Cancer happens when the gene p53 mutates. Probability of p53 to mutate per year is 5%.

How many years before a patient gets disease?

Which discrete probability distribution would you use to answer?

- A. Poisson
- B. Binomial
- C. Geometric
 - D. Negative Binomial
 - E. I have no idea

Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2,, 0 < \lambda$
Binomial	$\binom{n}{x} p^x (1-p)^{n-x}$
	$x = 0, 1,, n, 0 \le p \le 1$
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	$x = 1, 2,, 0 \le p \le 1$
Negative binomial	$\binom{x-1}{r-1} (1-p)^{x-r} p^r$
	$x = r, r + 1, r + 2,, 0 \le p \le 1$

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Continuous Probability Distributions

- **2. (8 points)** The length of stay at a specific emergency department in Phoenix, Arizona, in 2009 had a mean of 4.6 hours with a standard deviation of 2.9. Assume that the length of stay is normally distributed.
 - **(A) (4 points)** What is the probability of a length of stay greater than 10 hours?

(B) (4 points) How long does one have to stay in this emergency room to know that approximately 25% of all visits last even longer?

- **2. (8 points)** The length of stay at a specific emergency department in Phoenix, Arizona, in 2009 had a mean of 4.6 hours with a standard deviation of 2.9. Assume that the length of stay is normally distributed.
 - **(A) (4 points)** What is the probability of a length of stay greater than 10 hours?

Answer: (10-4.6)/2.9=1.86 Using table one finds Prob=1-0.9687=0.0313

(B) (4 points) How long does one have to stay in this emergency room to know that approximately 25% of all visits last even longer?

Answer: Using table one finds P(Z<0.67)=0.75 meaning it is 4.6+2.9*0.67=6.543

WHY DO WHALES JUMP & WHY ARE WITCHES GREEN WHY ARE THERE MIRRORS ABOVE BEDS WHY IS SEA SALL DELITED & SUMY ARE THERE TREES IN THE MIDDLE OF FIELDS & WHY IS THERE NOT A POKEMON MMO TO SERVE I AUGHING IN TV SHOWS WHY ARE THERE DOORS ON THE FREEWAY # 18 WHY AREN'T THERE ANY COUNTRIES IN ANTARCTICA WHY ARE THERE SCARY SOUNDS IN MINECRAFT WHY ISTHERE KICKING IN MY STOMACH WHY ARE THERE TWO SLASHES AFTER HTTP WHY ARE THERE CELEBRITIES, DO OYSTERS HAVE PEARLS WHY DO THEY CALL IT THE CLAP WHY ARE THE AVENGERS FIGHTING THE X MEN 5 WHY ARE KYLE AND CARTMAN FRIENDS WHY IS WOLVERINE NOT IN THE AVENGERS \$

WHY IS THERE AN ARROW ON AANG'S HEAD WHY ARE TEXT MESSAGES BLUE WHY ARE THERE MUSTACHES ON CLOTHES (

Credit: XKCD comics

WHY ARE THERE SLAVES IN

WHY DO TWINS HAVE DIFFERENT FINGERPRINTS & WHY IS HTTPS CROSSED OUT IN RED WHY ARE AMERICANS AFRAID OF DRAGONS WHY IS THERE A RED LINE THROUGH HTTPS ON FACEBOOK

SWHY ARE THERE SURPRIS OF CHATES AND SWHEET PHILEGARDS LAWY ARE THE CONTROLLED AND SWHEET PHILEGARD LAWY AND SWHEET PHIL

WHY ARE THERE

GHOSTS

₹WHY IS HTTPS IMPORTANT ONALL

WHY AREN'T MY ARMS GROWING

WHY ARE THERE SO MANY CROWS IN ROCHESTER, MIN

WHY IS THERE AN OWL OUTSIDE MY WINDOW

WHY ARE THERE MUSTACHES ON CARS I WHY IS EARTH TILTED & WHY ARE THERE MUSTACHES EVERYWHERE

WHY ARE THERE BRIDESMAIDS WHY ARE THERE TINY SPIDERS IN MY HOUSE
WHY DO DYING PEOPLE REACH UP WHY ARE THERE TINY SPIDERS IN MY HOUSE
WHY AREN'T THERE MARGOSE ARTERIES TO A MY AREN'T THERE WARROUSE ARTERIES TO A MY AREN'T THE WARROUSE ARTERIES TO A MY AREN'T THE WARROUSE ARTERIES TO A MY AREN'T THE WARROUSE AND THE WARROUSE AND T マWHY DO SPIDERS CON IS WHY ARE THERE HUGE SPIDERS IN MY HOUSE WHY ARE THERE

뉜 WHY ARE THERE LOTS OF SPIDERS IN MY HOUSE 包WHY ARE THERE SPIDERS IN MY ROOM AWHY ARE THERE SO MANY SPIDERS IN MY ROOM

DYING 50

 $\overline{m{\eta}}$ WHY 15 THERE NO GPS IN LAPTOPS $m{arepsilon}$ OWHY DO KNEES CLICK 子 WHY IS PROGRAMMING SO HARD WHY AREN'T THERE E. GRADES TO WHY IS THERE A O OHN RESISTER WHY AREN'T THERE E. GRADES TO WHY DO ANGERICANS HATE SOCCER WHY IS ISOLATION BAD WHY DO RHYMES SOUND GOOD WHY DO BOY'S LIKE ME WHY DON'T BOY'S LIKE ME WHY IS THERE NO SOUND ON CAN WHY IS THERE ALWAYS A JAVA UPDATE TO WHY AREN'T BULLETS SHARP WHY ARE THERE RED DOTS ON MY THIGHS WHY AREN'T BULLETS SHARP WHY IS LYING GOOD THE

WHY IS SEX **50 IMPORTANT** WHY IS THERE AN OWL ON THE DOLLAR BILL WHY ARE THERE TWO SPOCKS

YS WET S

WHY AREN'T MY QUAIL LAYING EGGS WHY ARE ULTRASOUNDS IMPORTANT WHY AREN'T MY QUAIL EGGS HATCHING WHY IS STEALING WRONG {idwhy aren't there any foreign military bases in america

WHY ARE CIGARETTES LEGAL WHY ARE THERE DUCKS IN MY POOL WHY IS JESUS WHITE G WHY DO Q TIPS FEEL GOOD Z



SQUIRRELS

WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE WHY IS THERE LIQUID IN MY EAR

> WHY AREN'T THERE GUNS IN HARRY POTTER

Two-sample hypothesis

Mating Calls. In a study of mating calls in the gray treefrogs *Hyla hrysoscelis* and *Hyla versicolor*, Gerhart (1994) reports that in a location in Lousiana the following data on the length of male advertisement calls have been collected:

	Sample	Average	SD of	Duration
	size	duration	duration	range
Hyla chrysoscelis	43	0.65	0.18	0.36-1.27
Hyla versicolor	12	0.54	0.14	0.36 - 0.75

The two species cannot be distinguished by external morphology, but *H. chrysoscelis* are diploids while *H. versicolor* are tetraploids. The triploid

crosses exhibit high mortality in larval stages, and if they attain sexual maturity, they are sterile. Females responding to the mating calls try to avoid mismatches.

Based on the data summaries provided, test whether the length of call is a discriminatory characteristic? Use $\alpha = 0.05$.

	Sample	Average	SD of
	size	duration	duration
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	Sample	Average	SD of
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Hyla versicolor	12	0.54	0.14

Based on the data summaries provided, test whether the length of call is a discriminatory characteristic? Use $\alpha = 0.05$.

- 1. Use two-sided hypothesis
- 2. z_{alpha/2}=1.96
- 3. Z=(0.65-0.54)/sqrt(0.18.^2/43+0.14.^2/12)=2.2516
- 4. Since Z> z_{alpha/2} null hypothesis can be rejected

Continuous probability distributions

Equipment Aging. Suppose that the lifetime T of a particular piece of laboratory equipment (in 1000 hour units) is an exponentially distributed random variable such that $\mathbb{P}(T > 10) = 0.8$.

- (a) Find the "rate" parameter, λ .
- (b) What are the mean and standard deviation of the random variable T?

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- (a) Find the "rate" parameter, λ .
- (b) What are the mean and standard deviation of the random variable T?

PDF:
$$P(t) = \lambda exp(-\lambda t)$$

Inverse $CDF = P(T>t) = exp(-\lambda t)$
 $-\lambda \cdot 10 = ln(0.8)$
 $\lambda = 0.0223$