# BIOE 582 Midterm Exam

**Name:**________________________

1. **(10 points)** A random sample of size $n_1 = 16$ is selected from a normal population with a mean of 80 and a standard deviation of 12.
   **(A)** Find the probability that the sample mean $\overline{X_1}$ is less than 70.

   **Answer:**

   $P(\bar{X}_1 < 70) = P\left(\frac{\bar{X}_1 - 80}{12/\sqrt{16}} < -3.33\right) = 0.000429$

   **(B)** Now a second random sample of size $n_2 = 25$ is taken from another normal population with mean 70 and standard deviation 10. Let $\overline{X_1}$ and $\overline{X_2}$ be the two sample means. Find the probability that $\overline{X_1} - \overline{X_2}$ exceeds 6.

   **Answer:** Let $\Delta X = \overline{X_1} - \overline{X_2}$ denote the sample difference, which approximately follows normal distribution with mean equal to 80-70=10 and standard deviation $\sigma = \sqrt{\frac{12^2}{16} + \frac{10^2}{25}} = 3.606$. Using z-table, we could find $P(\Delta X > 6) = 1 - P\left(\frac{\Delta X - 10}{3.606} < 1.109\right) = 0.866$

2. **(10 points)** A tobacco company collected and analyzed a sample of 36 cigarettes and found the sample mean to be 1.7 mg. It is known that the standard deviation of cigarette's nicotine content is 0.3 mg. To comply with regulations the average content of this type of cigarettes has to be below 1.6 mg.
   **(A)** Formulate null and alternative hypothesis regarding the company compliance with regulations and compute its P-value.

   **Answer**: One sides test $\begin{Bmatrix} H_0 : \mu <= 1.6 \\ H_1 : \mu > 1.6 \end{Bmatrix}$. The z-statistic is $z^* = \frac{1.7 - 1.6}{0.3/6} = 2$.

   $P$ value = $P(Z > 2) = 0.0228$

   **(B)** Could you reject the null hypothesis that company complies with regulations at 1% significance level?

   **Answer**: No since P-value is >0.01

3. **(10 points)** The operations manager of a large production plant would like to estimate the mean amount of time a worker takes to assemble a new electronic component. Assume that the standard deviation of this assembly time is 3.6 minutes. After observing a sample of 100 workers assembling similar devices, the manager noticed that their average time was 16.2 minutes. Construct a 90% two-sided confidence interval for the population mean of the assembly time.

**Answer:** Let $\mu$ denote the mean assembly time (in minutes). We want a 90% confidence interval for $\mu$ based on the following information: $n = 100$, $\bar{X} = 16.2$, $\alpha$=0.1, $\sigma$=3.6. Since $\sigma$ is known, we can use normal distribution to calculate confidence interval:

$$\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 16.2 \pm (1.65)\frac{3.6}{10} = [15.61, 16.79]$$

4. **(10 points)** A novel biomarker assay is 95% effective in detecting a certain disease when the patient is sick. The test also yields 2% false-positive result, that is to say, test reports a disease when the patient is not sick. If 1% of the population has the disease, what is the probability that a randomly chosen member of the population is sick if the biomarker was positive?

**Answer:** Let us denote Y/N as the presence/absence of the disease and +/- denote positive/negative diagnosis result.

P(Y|+) = P(+|Y)P(Y)/P(+) = P(+|Y)P(Y)/(P(+|Y)P(Y) + P(+|N)P(N)) = 0.95*0.01/(0.95*0.01+0.02*0.99) = 0.3242

5. **(10 points)** The joint probability mass function of discrete random variables X and Y taking values x = 1, 2 and y = 1, 2 respectively, is given by $f_{XY}(x,y)$ = c*x*y. Find the normalization constant c, the marginal probability $f_X$(X=1) and conditional probability distribution that Y=2 given that X = 1

**Answer:**
c(1+2+2+4) = 1, so c = 1/9 = 0.1111
$f_X$(1) = c*(1+2) = 1/3 = 0.3333
P(Y=2|X=1) = P(X=1, Y=2)/P(X=1) = (2/9)/(1/3) = 2/3 = 0.6666

6. **(20 points)** You are doing a long series of experiments. Assume that each of your experiments has a probability of 0.02 of succeeding. Assume that your experiments are independent from each other.

   **(A)** What is the probability that you first succeed on tenth experiment?

   P(X=10) = $(1-0.02)^9$ *0.02 = 0.0167

   **(B)** What is the probability that it requires more than five experiments for you to succeed?

$P(X > 5) = 1 - P(X=1) - P(X=2) - P(X=3) - P(X=4) - P(X=5)$
$=1 - 0.98^0 *0.02 - 0.98^1 *0.02 - 0.98^2 *0.02 - 0.98^3 *0.02 - 0.98^4 *0.02 = 0.9039$
Easier solution: $P(X>5)=0.98^5=0.9039$

**(C)** What is the average number of experiments you need to perform to succeed twice?

This X follows negative binomial distribution, the mean value of X is $2/0.02 = 100$.

**(D)** What is the probability that the second time you succeeded was on the tenth experiment since you started?

Probability based on negative binomial distribution $= 9*0.02^2*0.98^8 = 0.0031$