

Did you know that M&M's® Milk Chocolate Candies are supposed to come in the following percentages: 24% blue, 20% orange, 16% green, 14% yellow, 13% red, 13% brown?



How large is a sample needed for 95% CI on the percentage of blue M&Ms to be less than +/- 4%

Same question for red M&Ms?

For blue M&Ms $p = 0.24$

$$1.96 \sqrt{\frac{0.24(1-0.24)}{n}} < 0.04$$

$$n > \left(\frac{1.96}{0.04}\right)^2 0.24 \times (1-0.24) = 438 \text{ M\&Ms or}$$

~ 2 x 7oz bags with 210 candies each

For red M&Ms $p = 0.13$

$$n > \left(\frac{1.96}{0.04}\right)^2 \times 0.13 \times (1-0.13) \approx 271 \text{ M\&Ms or}$$

~ 1 x 7oz bag

424

Example 10-7

Chocolate and Cardiovascular Health

An article in *Nature* (2003, Vol. 424, p. 1013) described an

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Plasma antioxidants from chocolate

Dark chocolate may offer its consumers health benefits the milk variety cannot match.

There is some speculation that dietary flavonoids from chocolate, in particular (-)-epicatechin, may promote cardiovascular health as a result of direct antioxidant effects or through antithrombotic mechanisms¹⁻³. Here we show that consumption of plain, dark chocolate (Fig. 1) results in an increase in both the total antioxidant capacity and the (-)-epicatechin content of blood plasma, but that these effects are markedly reduced when the chocolate is consumed with milk or if milk is incorporated as milk chocolate. Our findings indicate that milk may interfere with the absorption of antioxidants from chocolate *in vivo* and may therefore negate the potential health benefits that can be derived from eating moderate amounts of dark chocolate.

To determine the antioxidant content of different chocolate varieties, we took dark chocolate and milk chocolate prepared from the same batch of cocoa beans and defatted them twice with *n*-hexane before extracting them with a mixture of water, acetone and acetic acid (70.0:29.8:0.2 by volume). We measured their *in vitro* total antioxidant capacities using the ferric-reducing antioxidant potential (FRAP) assay⁴; FRAP

reduced iron per 100 g for dark and milk chocolate, respectively. Volunteers must therefore consume twice as much milk chocolate as dark chocolate to receive a similar intake of antioxidants.

We recruited 12 healthy volunteers (7 women and 5 men with an average age of 32.2 ± 1.0 years (range, 25–35 years). Subjects were non-smokers, had normal blood lipid levels, were taking no drugs or vitamin supplements, and had an average weight of 65.8 ± 3.1 kg (range, 46.0–86.0 kg) and body-mass index of 21.9 ± 0.4 kg m⁻² (range, 18.6–23.6 kg m⁻²). On different days, following a crossover experimental design, subjects consumed 100 g dark chocolate, 100 g dark chocolate with 200 ml full-fat milk, or 200 g milk chocolate (containing the equivalent of up to 40 ml milk).

One hour after subjects had ingested the chocolate, or chocolate and milk, we measured the total antioxidant capacity of their plasma by FRAP assay. Plasma antioxidant levels increased significantly after consumption of dark chocolate alone, from 100 ± 3.5% to 118.4 ± 3.5% (*t*-test, *P* < 0.001), returning to baseline values (95.4 ± 3.6%) after 4 h (Fig. 2a). There was



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Figure 1 Stack of benefits? Unlike its milky counterpart, dark chocolate may provide more than just a treat for the tastebuds.

could be due to the formation of secondary bonds between chocolate flavonoids and milk proteins^{6,7}, which would reduce the biological accessibility of the flavonoids and therefore the chocolate's potential antioxidant properties *in vivo*.

Our findings highlight the possibility

Vol. 424
↓

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Sweet matlab exercise #1

- Download **dark_vs_milk_chocolate_analysis_template.m** at the course website. **Correct all ??** In the file
- **dark=[118.8 122.6 115.6 113.6 119.5 115.9 115.8 115.1 116.9 115.4 115.6 107.9];**
- **milk=[102.1 105.8 99.6 102.7 98.8 100.9 102.8 98.7 94.7 97.8 99.7 98.6]**
- Use Z-statistics to calculate **P-value** of the null hypothesis **H₀** that **milk = dark** against **H₁** that **dark > milk**. **P_value_z=1-normcdf(Z)**
- Repeat using T-statistics. # of degrees of freedom is **dof=2*(n-1)**
P_value_t=tcdf(T, dof)

Sweet matlab exercise #1

- `dark=[118.8 122.6 115.6 113.6 119.5 115.9 115.8 115.1 116.9 115.4 115.6 107.9];`
- `milk=[102.1 105.8 99.6 102.7 98.8 100.9 102.8 98.7 94.7 97.8 99.7 98.6]`
- `x_dark=mean(dark) % sample mean dark chocolate`
- `x_milk=mean(milk) % sample mean milk chocolate`
- `s_dark=std(dark) % sample std dark chocolate`
- `s_milk=std(milk) % sample std milk chocolate`
- `n=12 % sample size of both dark and milk`
- `std_xdiff=sqrt(s_dark.^2./2+s_milk.^2./n) % std diff x`
- `z_stat=(x_dark-x_milk)./std_xdiff % z-statistic`
- `P_value_z=erfc(z_stat./sqrt(2))./2 % P-value of null true`
- `% P_value_z=9.9629e-34`
- `dof=(n-1)+(n-1) % # of degrees of freedom`
- `P_value_t=tcdf(z_stat,dof,'upper') % P-value of null true`
- `%P_value_t= 1.8417e-11`

Credit: XKCD
comics

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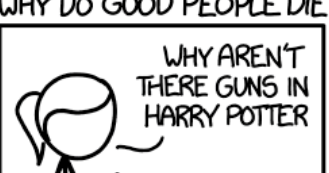


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Goodness of Fit
hypothesis testing:

Pearson's chi-square test

Did you know that M&M's® Milk Chocolate Candies are supposed to come in the following percentages: 24% blue, 20% orange, 16% green, 14% yellow, 13% red, 13% brown?

<http://www.scientificameriken.com/candy5.asp>

“To our surprise M&Ms met our demand to review their procedures in determining candy ratios. It is, however, noted that the figures presented in their email differ from the information provided from their website (<http://us.mms.com/us/about/products/milkchocolate/>). An email was sent back informing them of this fact. To which M&Ms corrected themselves with one last email:

In response to your email regarding M&M'S CHOCOLATE CANDIES

Thank you for your email.

On average, our new mix of colors for M&M'S® Chocolate Candies is:

M&M'S® Milk Chocolate: 24% blue, 20% orange, 16% green, 14% yellow, 13% red, 13% brown.

M&M'S® Peanut: 23% blue, 23% orange, 15% green, 15% yellow, 12% red, 12% brown.

M&M'S® Kids MINIS®: 25% blue, 25% orange, 12% green, 13% yellow, 12% red, 13% brown.

M&M'S® Crispy: 17% blue, 16% orange, 16% green, 17% yellow, 17% red, 17% brown.

M&M'S® Peanut Butter and Almond: 20% blue, 20% orange, 20% green, 20% yellow, 10% red, 10% brown.

Have a great day!

Your Friends at Masterfoods USA
A Division of Mars, Incorporated



How to accept or reject the null hypothesis that these probabilities are correct from a finite sample?

Pearson χ^2 Goodness of Fit Test

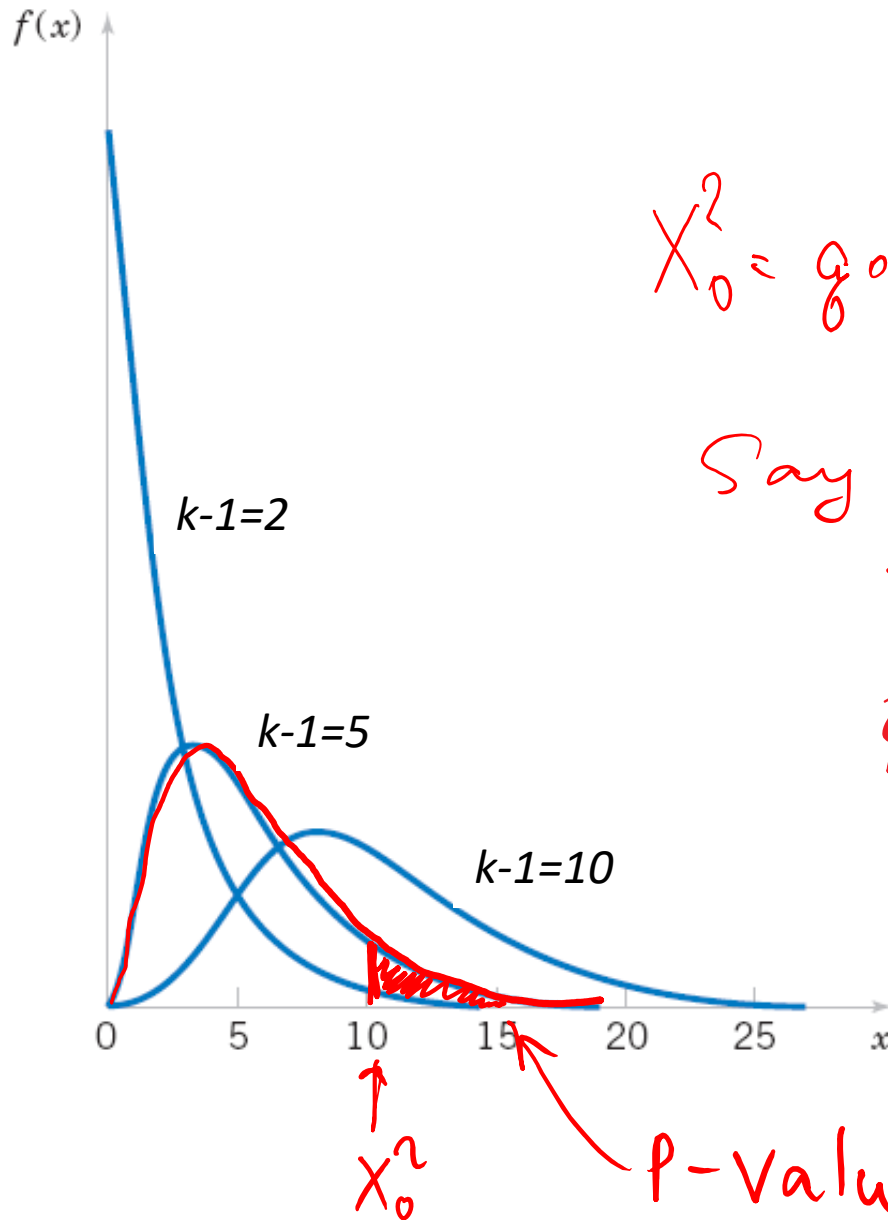
- Assume there is a **sample of size n** from a population with **k classes** (e.g. 6 M&M colors)
- **Null hypothesis** H_0 : class i has frequency f_i in the population
- **Alternative hypothesis** H_1 : some population frequencies are inconsistent with f_i
- Let O_i be the **observed number** of sample elements in the i th class and $E_i = n f_i$ be the **expected number** of sample elements in the i th class.
- **Group any bin** with $E_i < 3$ with
 - a) if numerical value of i is important, group it with its neighbor ($k=i-1$ or $k=i+1$) which has the smallest E_k until $E_{group} \geq 3$;
 - b) If numerical value of i is irrelevant, group together all $E_i < 3$ bins until $E_{group} \geq 3$
- The **test statistic** is

$$X_0^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i} \quad (9-47)$$

P-value is calculated based on the **chi-square distribution** with **$k-1$ degrees of freedom**:

$$\text{P-value} = \text{Prob}(H_0 \text{ is correct}) = 1 - \text{CDF_chi-squared}(X_0^2, k-1)$$

chi² Goodness of Fit Test is a one-sided hypothesis



$$X_0^2 = \text{gof} = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}$$

Say $X_0^2 = 10$

For M&M

$$k = 6 \rightarrow k-1 = 5$$

X_0^2 p-value that null hypothesis is correct

M&M group exercise

- **DO NOT EAT CANDY BEFORE COUNTING IS FINISHED!**
THEN, PLEASE, DO.
- We will be testing three null hypotheses one after another:
 - M&M peanut official data: 23% blue, 23% orange, 15% green, 15% yellow, 12% red, 12% brown
 - Website (fan collected) data from <http://joshmadison.com/2007/12/02/mms-color-distribution-analysis>:
18.36% blue, 20.76% orange, 18.44% green, 14.08% yellow, 14.20% red, 14.16% brown
 - Uniform distribution: 1/6~16.67% of each candy color
- You will estimate P-values for each one of these null hypotheses
- Hints: O_i – is the observed # of candies of color i ;
calculate the expected # $E_i = (\# \text{ candies in your sample}) * f_i$

Use **1-chi2cdf(X0squared, 5)** for P-value

$$X_0^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}$$

M&M matlab exercise

- `observed=mm_table(group,:); group % use when analyzing one group`
- `f_mm=[0.24,0.2,0.16, 0.14, 0.13,0.13];`
- `f_u=1./6.*ones(1,6);`
- `f_website=[18,21,18,14,14,14,14];`
- `f_website=f_website./sum(f_website);`
- `%p_website=[0.1836, 0.2076, 0.1844, 0.1408, 0.1420, 0.1416]`
- `%p_u=[0.1500, 0.2200, 0.2100, 0.1200, 0.1600, 0.1500];`
- `n=sum(observed)`
- `expected_u=n.*f_u;`
- `expected_mm=n.*f_mm;`
- `expected_website=n.*f_website;`
- `gf_mm=0; gf_u=0; gf_website=0;`
- `for m=1:6;`
- `gf_mm=gf_mm+(observed(m)...`
- `-expected_mm(m)).^2./expected_mm(m);`
- `gf_u=gf_u+(observed(m)-expected_u(m)).^2./expected_u(m);`
- `gf_website=gf_website+(observed(m)...`
- `-expected_website(m)).^2./expected_website(m);`
- `end;`
- `disp('goodness of fit of MM ='); disp(num2str(gf_mm));`
- `disp('p-value of MM ='); disp(num2str(1-chi2cdf(gf_mm,5))); disp(' ');`
- `disp('goodness of fit of website ='); disp(num2str(gf_website));`
- `disp('p-value of MM ='); disp(num2str(1-chi2cdf(gf_website,5))); disp(' ');`
- `disp('goodness of fit of uniform ='); disp(num2str(gf_u));`
- `disp('p-value of uniform='); disp(num2str(1-chi2cdf(gf_u,5)));`

	blue	orange	green	yellow	red	brown	P-value official	P-value uniform	P-value weaker
Group 1 (Milk)	39	43	33	22	30	26			
Group 2 (Milk)	36	34	20	24	22	28	0.393	0.345	0.147
Group 3 (Dark)	39	41	20	17	18	25	0.28	0.0065	0.065
Group 4 (Dark)	32	31	18	11	24	29			

Groups 1 and 2 – Milk chocolate

They accept all three hypotheses with uniform being weaker (smaller P-value but >0.05)

Groups 3 and 4 – Dark chocolate

They reject all three hypotheses except group 3 confirming the official hypothesis

Statistical tests of independence

How to test the hypothesis if multiple samples are drawn from the same population?

- Table: **samples (Student groups) – rows**, **classes (M&M colors) – columns**
- Test if color fractions are independent from group
- **$P(\text{Group 1 and Color = green}) = P(\text{Group 1}) * P(\text{Color green})$**
- Compute for all groups/colors $6 * 4 = 24$ in our case

$$E_{\text{green}}(\text{group 1}) = n_{\text{tot}} * (\text{group 1} / n_{\text{tot}}) * (\text{green} / n_{\text{tot}})$$

- $$\chi^2 = \sum_{\text{groups \& colors}}^{n_{\text{tot}}} \frac{(O_{\text{color}}(\text{group}) - E_{\text{color}}(\text{group}))^2}{E_{\text{color}}(\text{group})}$$
- # degrees of freedom = **(colors-1) * (groups-1)**

- M&M exercise Fall 2025
- Was the Milk M&M bag mixed well?
Let's compare the data from groups 1 and 2
- Was the Dark M&M bag mixed well?
Let's compare the data from groups 3 and 4

	Blue	Orange	Green	Yellow	Red	Brown
Group 1 (milk)	39	43	33	22	30	26
Group 2 (milk)	36	34	20	24	22	28
Group 3 (dark)	39	41	20	17	18	25
Group 4 (dark)	32	31	18	11	24	29

- Using $\chi^2 = \sum_{groups \& \ colors} \frac{(O_{color}(group) - E_{color}(group))^2}{E_{color}(group)}$

with # degrees of freedom (colors-1)*(groups-1)

Find P-value of null hypothesis H_0 that samples are independent from each other

- For milk the P-value is 0.6356. I mixed this bag well!
- For dark the P-value is 0.5646. This bag was already mixed

Was the bag of milk M&Ms well mixed?

- `clear mm_table`
- `mm_table=mm_table_all(1:2,:);`
- `ngroups=2;`
- `ncolors=6;`
- `sumt=sum(sum(mm_table))`
- `sum_color=sum(mm_table, 1)`
- `sum_group=sum(mm_table, 2)`
- `mm_exp=kron(sum_group,sum_color)./sumt`
- `gof=sum(sum((mm_table-mm_exp).^2./mm_exp))`
- `P_value_gof=1-chi2cdf(gof, (ngroups-1)*(ncolors-1))`
- **`%gof = 3.4192`**
- **`P_value_gof = 0.6356`**
- **The null model that samples are independent is not rejected → The bag was well mixed!**

Batch effect

Does color composition vary between

Milk and Dark chocolate M&M

- Milk total: groups 1+2

75 77 53 46 52 54

- Dark chocolate: groups 3+4

71 72 38 28 42 54

- Test if they are significantly different from each other:

- Same statistical independence test:

ngroups=2; ncolors=6;

- Results:

Goodness of Fit = 4.1330

P_value_gof = 0.5304

- Batch effect is **not statistically significant. Milk and Dark M&Ms have the same color frequencies**

Credit: XKCD
comics

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Goodness of fit with a PDF defined by **m** parameters

- As before: **k** classes (e.g. M&M colors)
- Use **parameter estimators** to find **the best parameters** for the fit
 - Method of moments
 - MLE: method of maximum likelihood
- Use chi-squared distribution with **k-1-m** degrees of freedom
- As before: if $E_i < 3$, group it together with another group and reduce **k** by 1

$$X_0^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i} \quad (9-47)$$

9-7 Testing for Goodness of Fit

Example 9-12

EXAMPLE 9-12 Printed Circuit Board Defects Poisson Distribution

The number of defects in printed circuit boards is hypothesized to follow a Poisson distribution. A random sample of $n = 60$ printed boards has been collected, and the following number of defects observed.

Number of Defects	Observed Frequency
0	32
1	15
2	9
3	4

9-7 Testing for Goodness of Fit

Example 9-12

The mean of the assumed Poisson distribution in this example is unknown and must be estimated from the sample data. The estimate of the mean number of defects per board is the sample average, that is, $(32 \cdot 0 + 15 \cdot 1 + 9 \cdot 2 + 4 \cdot 3) / 60 = 0.75$. From the Poisson distribution with parameter 0.75, we may compute p_i , the theoretical, hypothesized probability associated with the i th class interval. Since each class interval corresponds to a particular number of defects, we may find the p_i as follows:

$$p_1 = P(X = 0) = \frac{e^{-0.75}(0.75)^0}{0!} = 0.472$$

$$p_2 = P(X = 1) = \frac{e^{-0.75}(0.75)^1}{1!} = 0.354$$

$$p_3 = P(X = 2) = \frac{e^{-0.75}(0.75)^2}{2!} = 0.133$$

$$p_4 = P(X \geq 3) = 1 - (p_1 + p_2 + p_3) = 0.041$$

9-7 Testing for Goodness of Fit

Example 9-12

The expected frequencies are computed by multiplying the sample size $n = 60$ times the probabilities p_i . That is, $E_i = np_i$. The expected frequencies follow:

Number of Defects	Probability	Expected Frequency
0	0.472	28.32
1	0.354	21.24
2	0.133	7.98
3 (or more)	0.041	2.46

9-7 Testing for Goodness of Fit

Example 9-12

Since the expected frequency in the last cell is less than 3, we combine the last two cells:

Number of Defects	Observed Frequency	Expected Frequency
0	32	28.32
1	15	21.24
2 (or more)	13	10.44

The chi-square test statistic in Equation 9-47 will have $k - p - 1 = 3 - 1 - 1 = 1$ degree of freedom, because the mean of the Poisson distribution was estimated from the data.

9-7 Testing for Goodness of Fit

Example 9-12

The seven-step hypothesis-testing procedure may now be applied, using $\alpha = 0.05$, as follows:

1. **Parameter of interest:** The variable of interest is the form of the distribution of defects in printed circuit boards.
2. **Null hypothesis:** H_0 : The form of the distribution of defects is Poisson.
3. **Alternative hypothesis:** H_1 : The form of the distribution of defects is not Poisson.
4. **Test statistic:** The test statistic is

$$\chi_0^2 = \sum_{i=1}^k \frac{(o_i - E_i)^2}{E_i}$$

9-7 Testing for Goodness of Fit

Example 9-12

5. **Reject H_0 if:** Reject H_0 if the P -value is less than 0.05.

6. **Computations:**

$$\chi_0^2 = \frac{(32 - 28.32)^2}{28.32} + \frac{(15 - 21.24)^2}{21.24} + \frac{(13 - 10.44)^2}{10.44} = 2.94$$

7. **Conclusions:** We find from Appendix Table III that $\chi_{0.10,1}^2 = 2.71$ and $\chi_{0.05,1}^2 = 3.84$. Because $\chi_0^2 = 2.94$ lies between these values, we conclude that the P -value is between 0.05 and 0.10. Therefore, since the P -value exceeds 0.05 we are unable to reject the null hypothesis that the distribution of defects in printed circuit boards is Poisson. The exact P -value computed from Minitab is 0.0864.

Credit: XKCD
comics

WHY ARE THERE SLAVES IN THE BIBLE

WHY DO TWINS HAVE DIFFERENT FINGERPRINTS
WHY ARE AMERICANS AFRAID OF DRAGONS

WHY IS HTTPS CROSSED OUT IN RED
WHY IS THERE A LINE THROUGH HTTPS
WHY IS THERE A RED LINE THROUGH HTTPS ON FACEBOOK
WHY IS HTTPS IMPORTANT

QUESTIONS

FOUND IN GOOGLE AUTOCOMPLETE



WHY ARE THERE WEEKS
WHY DO I FEEL DIZZY

WHY AREN'T ECONOMISTS RICH

WHY ARE THERE SO MANY CROWS IN ROCHESTER, MN
WHY IS THERE PHLEGM

WHY DO AMERICANS CALL IT SOCCER

WHY IS PSYCHIC WEAK TO BUG

WHY ARE MY EARS RINGING

WHY DO CHILDREN GET CANCER

WHY ARE THERE SO MANY AVENGERS

WHY IS POSEIDON ANGRY WITH ODYSSEUS

WHY ARE THE AVENGERS FIGHTING THE X MEN

WHY IS THERE ICE IN SPACE

WHY IS WOLVERINE NOT IN THE AVENGERS

WHY ARE THERE ANTS IN MY LAPTOP

WHY IS EARTH TILTED
WHY IS SPACE BLACK
WHY IS OUTER SPACE SO COLD
WHY ARE THERE PYRAMIDS ON THE MOON
WHY IS NASA SHUTTING DOWN



WHY IS THERE AN OWL IN MY BACKYARD
WHY IS THERE AN OWL OUTSIDE MY WINDOW
WHY IS THERE AN OWL ON THE DOLLAR BILL
WHY DO OWLS ATTACK PEOPLE
WHY ARE AK 47s SO EXPENSIVE
WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE
WHY ARE THERE GODS
WHY ARE THERE TWO SPOCKS

WHY ARE DOGS AFRAID OF FIREWORKS
WHY IS THERE NO KING IN ENGLAND

WHY ARE THERE TINY SPIDERS IN MY HOUSE

WHY DO SPIDERS COME INSIDE

WHY ARE THERE HUGE SPIDERS IN MY HOUSE

WHY ARE THERE LOTS OF SPIDERS IN MY HOUSE

WHY ARE THERE SPIDERS IN MY ROOM

WHY ARE THERE SO MANY SPIDERS IN MY ROOM

WHY DO SPIDER BITES ITCH

WHY IS DYING SO SCARY

WHY IS THERE NO GPS IN LAPTOPS

WHY DO KNEES CLICK
WHY AREN'T THERE E GRADES
WHY IS ISOLATION BAD
WHY DO BOYS LIKE ME
WHY DON'T BOYS LIKE ME
WHY IS THERE ALWAYS A JAVA UPDATE
WHY ARE THERE RED DOTS ON MY THIGHS



WHY DO WHALES JUMP
WHY ARE WITCHES GREEN
WHY ARE THERE MIRRORS ABOVE BEDS
WHY DO I SAY UH
WHY IS SEA SALT BETTER
WHY ARE THERE TREES IN THE MIDDLE OF FIELDS
WHY IS THERE NOT A POKEMON MMO
WHY IS THERE LAUGHING IN TV SHOWS
WHY ARE THERE DOORS ON THE FREEWAY
WHY ARE THERE SO MANY SVCHOST.EXE RUNNING
WHY AREN'T THERE ANY COUNTRIES IN ANTARCTICA
WHY ARE THERE SCARY SOUNDS IN MINECRAFT
WHY IS THERE KICKING IN MY STOMACH
WHY ARE THERE TWO SLASHES AFTER HTTP
WHY ARE THERE CELEBRITIES
WHY DO SNAKES EXIST
WHY DO OYSTERS HAVE PEARLS
WHY ARE DUCKS CALLED DUCKS
WHY DO THEY CALL IT THE CLAP
WHY ARE KYLE AND CARTMAN FRIENDS
WHY IS THERE AN ARROW ON AANG'S HEAD
WHY ARE TEXT MESSAGES BLUE
WHY ARE THERE MUSTACHES ON CLOTHES
WHY ARE THERE MUSTACHES ON CARS
WHY ARE THERE MUSTACHES EVERYWHERE
WHY ARE THERE SO MANY BIRDS IN OHIO
WHY IS THERE SO MUCH RAIN IN OHIO
WHY IS OHIO WEATHER SO WEIRD
WHY ARE THERE MALE AND FEMALE BIKES

WHY AREN'T THERE DINOSAUR GHOSTS

WHY ARE THERE BRIDESMAIDS
WHY DO DYING PEOPLE REACH UP
WHY AREN'T THERE VARICOSE ARTERIES
WHY ARE OLD KUNGONS DIFFERENT

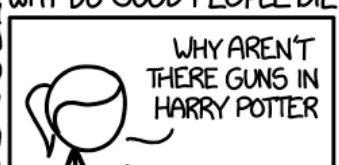


WHY IS PROGRAMMING SO HARD
WHY IS THERE A 0 OHM RESISTOR
WHY DO AMERICANS HATE SOCCER
WHY DO RHYMES SOUND GOOD
WHY DO TREES DIE
WHY IS THERE NO SOUND ON CNN
WHY AREN'T POKEMON REAL
WHY AREN'T BULLETS SHARP
WHY DO DREAMS SEEM SO REAL

WHY IS THERE HELL IF GOD FORGIVES

WHY IS GPS FREE

WHY IS LIFE SO BORING



WHY ARE ULTRASOUNDS IMPORTANT
WHY ARE ULTRASOUND MACHINES EXPENSIVE
WHY IS STEALING WRONG
WHY AREN'T THERE ANY FOREIGN MILITARY BASES IN AMERICA