

Reminder

Statistically independent events

Always true: $P(A \cap B) = P(A | B) \cdot P(B) = P(B | A) \cdot P(A)$

■ Two events

Two events are **independent** if **any one** of the following equivalent statements is true:

(1) $P(A|B) = P(A)$

(2) $P(B|A) = P(B)$

(3) $P(A \cap B) = P(A)P(B)$

■ Multiple events

The events E_1, E_2, \dots, E_n are independent if and only if for any subset of these events $E_{i_1}, E_{i_2}, \dots, E_{i_k}$,

$$P(E_{i_1} \cap E_{i_2} \cap \dots \cap E_{i_k}) = P(E_{i_1}) \times P(E_{i_2}) \times \dots \times P(E_{i_k})$$

Independence of Random Variables X and Y

- **Random variable independence** means that knowledge of **any** of the values of X **does not change** probabilities of **any** of the values of Y
- Opposite: **Dependence** implies that **some** values of X influence the probability of **some** values of Y

Independence for Discrete Random Variables

- Remember independence of events (slide 13 lecture 4) : Events are independent if **any one** of the three conditions are met:
 - 1) $P(A|B) = P(A \cap B)/P(B) = P(A)$ or
 - 2) $P(B|A) = P(A \cap B)/P(A) = P(B)$ or
 - 3) $P(A \cap B) = P(A) \cdot P(B)$
- Random variables independent if **all events** A that $Y=y$ and B that $X=x$ are independent if any one of these conditions is met:
 - 1) $P(Y=y | X=x) = P(Y=y)$ for any x or
 - 2) $P(X=x | Y=y) = P(X=x)$ for any y or
 - 3) $P(X=x, Y=y) = P(X=x) \cdot P(Y=y)$**for every pair x and y**

X and Y are Bernoulli variables

	Y=0	Y=1
X=0	2/6	1/6
X=1	2/6	1/6

Are they independent?

A. yes

B. no

C. I don't know

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X and Y are Bernoulli variables

	Y=0	Y=1
X=0	1/2	0
X=1	0	1/2

Are they independent?

A. yes

B. no

C. I don't know

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Credit: XKCD
comics

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Joint Probability Density Function Defined

The **joint probability density function** for the continuous random variables X and Y , denoted as $f_{XY}(x,y)$, satisfies the following properties:

(1) $f_{XY}(x,y) \geq 0$ for all x, y

(2)
$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{XY}(x,y) dx dy = 1$$

(3)
$$P((X,Y) \subset R) = \iint_R f_{XY}(x,y) dx dy \quad (5-2)$$

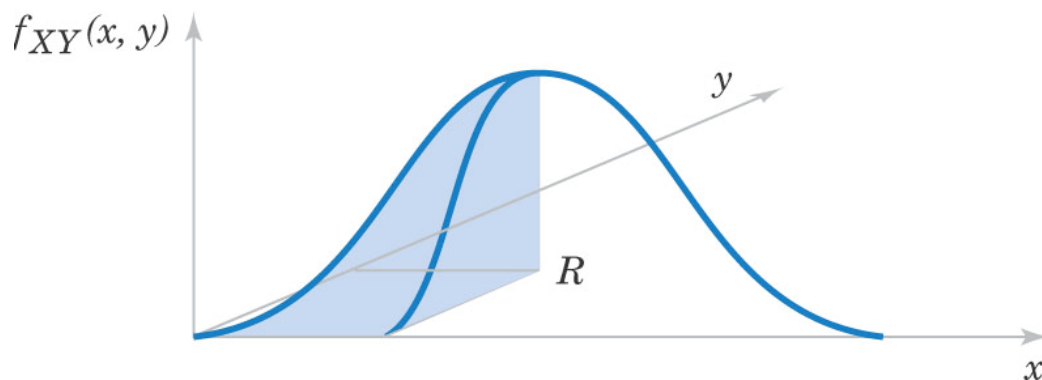


Figure 5-2 Joint probability density function for the random variables X and Y . Probability that (X, Y) is in the region R is determined by the **volume** of $f_{XY}(x,y)$ over the region R .

Joint Probability Density Function Graph

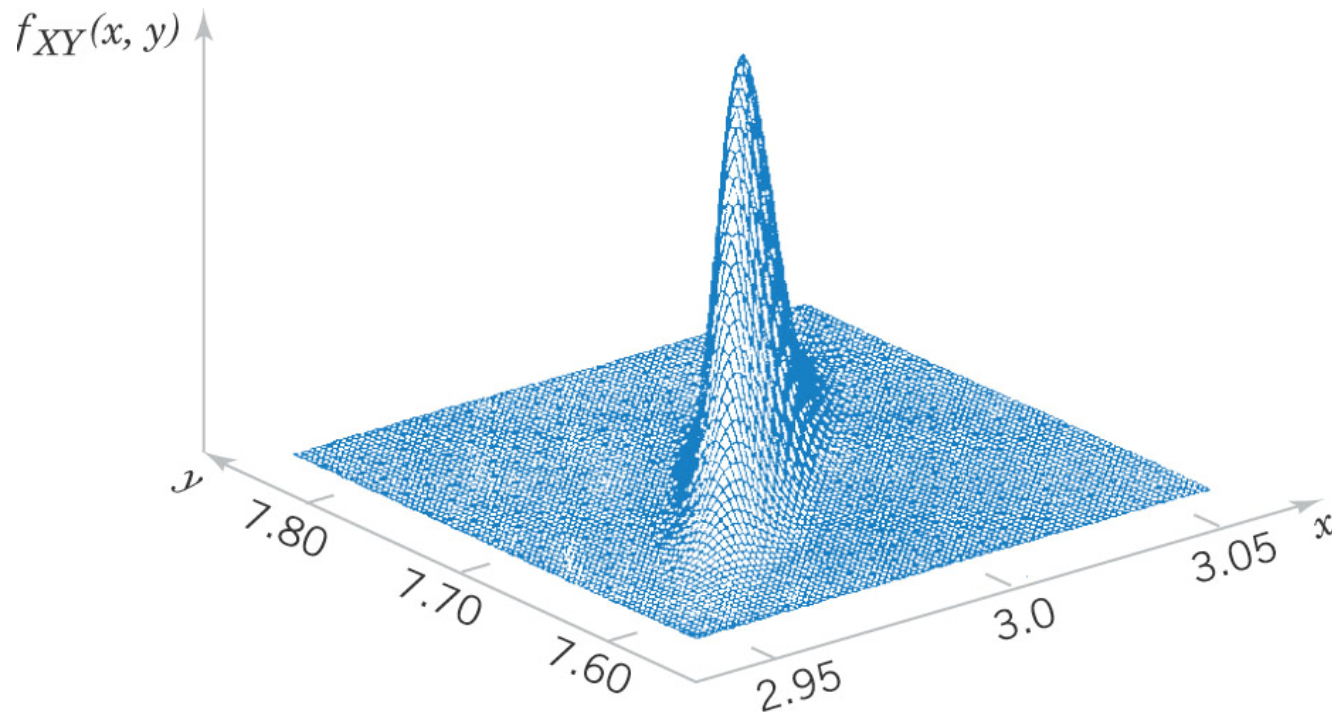


Figure 5-3 Joint probability density function for the continuous random variables X and Y of expression levels of two different genes. Note the asymmetric, narrow ridge shape of the PDF – indicating that small values in the X dimension are more likely to occur when small values in the Y dimension occur.

Marginal Probability Distributions (continuous)

- Rather than summing a discrete joint PMF, we integrate a continuous joint PDF.
- The marginal PDFs are used to make probability statements about one variable.
- If the joint probability density function of random variables X and Y is $f_{XY}(x,y)$, the marginal probability density functions of X and Y are:

$$f_X(x) = \int_y f_{XY}(x, y) dy$$

$$f_Y(y) = \int_x f_{XY}(x, y) dx \quad (5-3)$$

$$f_X(x) = \sum_y f_{XY}(x, y)$$

$$f_Y(y) = \sum_x f_{XY}(x, y)$$

Conditional Probability Density Function Defined

Given continuous random variables X and Y with joint probability density function $f_{XY}(x, y)$, the conditional probability density function of Y given $X=x$ is

$$f_{Y|x}(y) = \frac{f_{XY}(x, y)}{f_X(x)} = \frac{f_{XY}(x, y)}{\int_y f_{XY}(x, y) dy} \text{ if } f_X(x) > 0 \quad (5-4)$$

which satisfies the following properties:

(1) $f_{Y|x}(y) \geq 0$

(2) $\int f_{Y|x}(y) dy = 1$

(3) $P(Y \in B | X = x) = \int_B f_{Y|x}(y) dy$ for any set B in the range of Y

Compare to discrete: $P(Y=y | X=x) = f_{XY}(x, y) / f_X(x)$

Conditional Probability Distributions

- Conditional probability distributions can be developed for multiple random variables by extension of the ideas used for two random variables.
- Suppose $p = 5$ and we wish to find the distribution of X_1, X_2 and X_3 conditional on $X_4=x_4$ and $X_5=x_5$.

$$f_{X_1 X_2 X_3 | x_4 x_5}(x_1, x_2, x_3) = \frac{f_{X_1 X_2 X_3 X_4 X_5}(x_1, x_2, x_3, x_4, x_5)}{f_{X_4 X_5}(x_4, x_5)}$$

for $f_{X_4 X_5}(x_4, x_5) > 0$.

Independence for Continuous Random Variables

For random variables X and Y , if any one of the following properties is true, the others are also true. Then X and Y are **independent**.

(1) $f_{XY}(x, y) = f_X(x) \cdot f_Y(y)$

(2) $f_{Y|x}(y) = f_Y(y)$ for all x and y with $f_X(x) > 0$

(3) $f_{X|y}(x) = f_X(x)$ for all x and y with $f_Y(y) > 0$

(4) $P(X \in A, Y \in B) = P(X \in A) \cdot P(Y \in B)$ for any sets A and B in the range of X and Y , respectively. (5–7)

$P(Y=y|X=x)=P(Y=y)$ **for any x** or

$P(X=x|Y=y)=P(X=x)$ **for any y** or

$P(X=x, Y=y)=P(X=x) \cdot P(Y=y)$ **for any x and y**

Covariation, Correlations

Quick and dirty check for
linear (in)dependence
between variables

Covariance - 1 number to measure dependance between random variables

$\text{Cov}(X, Y)$ or σ_{xy}

$$\begin{aligned}\sigma_{xy} &= E[(X - \mu_x) \cdot (Y - \mu_y)] = \\ &= E(X \cdot Y) - \mu_x \cdot \mu_y\end{aligned}$$

- $\text{Var}(X) = \text{Cov}(X, X)$

- If X & Y are independent

$$\text{Cov}(X, Y) = E[X - \mu_x] \cdot E[Y - \mu_y] = 0$$

- $-\infty < \text{Cov}(X, Y) < +\infty$ Can be negative!

Covariance Defined

Covariance is a number quantifying the average *linear* dependence between two random variables.

The covariance between the random variables X and Y , denoted as $\text{cov}(X, Y)$ or σ_{XY} is

$$\sigma_{XY} = E[(X - \mu_X)(Y - \mu_Y)] = E(XY) - \mu_X\mu_Y$$

Montgomery, Runger 5th edition Eq. (5-14)

The units of σ_{XY} are the units of X times the units of Y .

Unlike the range of the variance, covariance can be negative: $-\infty < \sigma_{XY} < \infty$.

Covariance and PMF tables

y = number of times city name is stated	x = number of bars of signal strength		
	1	2	3
1	0.01	0.02	0.25
2	0.02	0.03	0.20
3	0.02	0.10	0.05
4	0.15	0.10	0.05

The probability distribution of Example 5-1 is shown.

By inspection, note that the **larger probabilities** occur as X and Y move in opposite directions. This indicates a **negative covariance**.

Covariance and Scatter Patterns

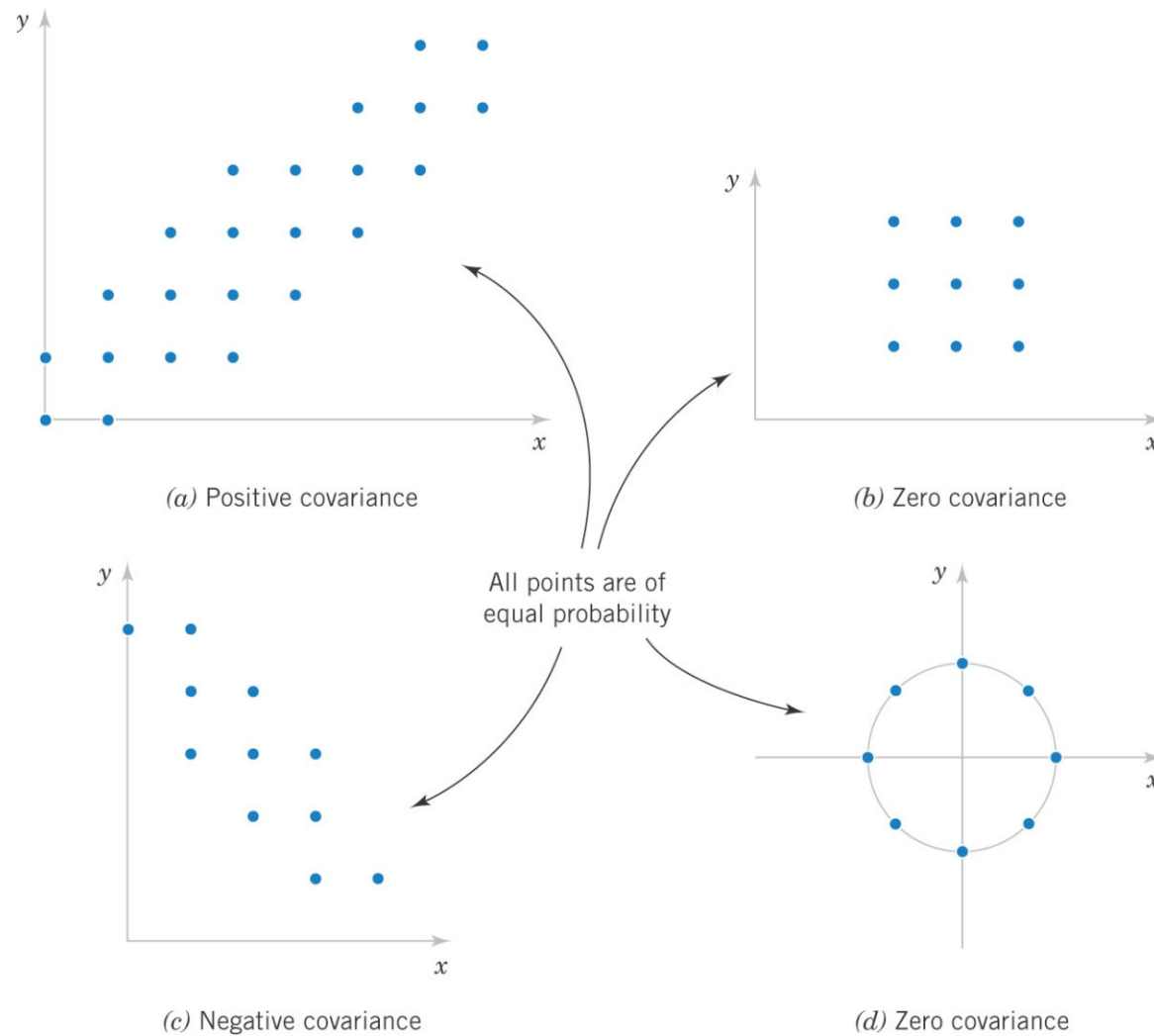
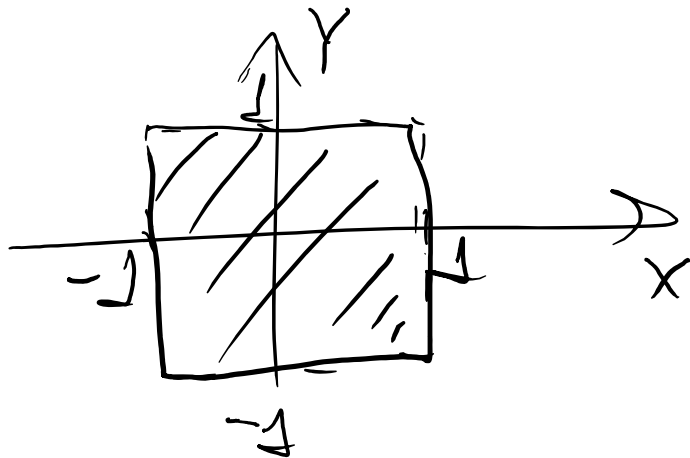


Figure 5-13 Joint probability distributions and the sign of $\text{cov}(X, Y)$. Note that covariance is a measure of linear relationship. Variables with non-zero covariance are **correlated**.

Example 1:

Uniform distribution in the square

$$-1 < X < 1, \quad -1 < Y < 1$$



$$\begin{cases} f_{X,Y}(x,y) = c & \text{if } -1 < x < 1 \text{ and } -1 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

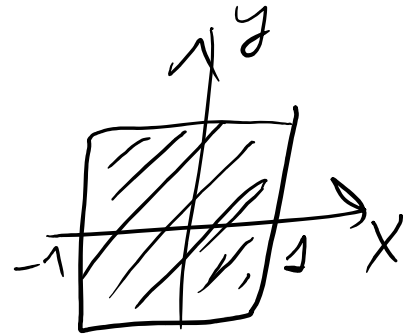
$$1 = \int_{\text{square}} dx dy f_{X,Y}(x,y) = c \cdot \text{Area} = c \cdot 4 \rightarrow c = \frac{1}{4}$$

Are X and Y independent? Yes they are

Let's test if $f_{XY}(x, y) = f_X(x) \cdot f_Y(y)$

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x, y) dy =$$

$$= \int_{-1}^1 \frac{1}{4} dy = \frac{1}{2} \text{ if } -1 < x < 1$$

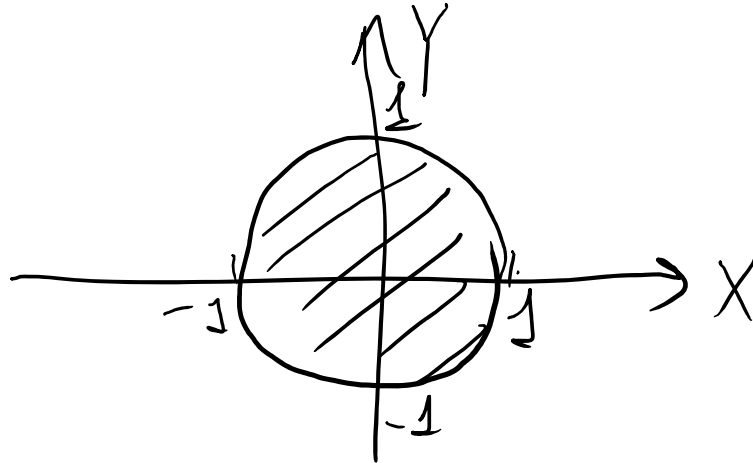


Same for $f_Y(y) = \frac{1}{2}$ if $-1 < y < 1$

$$\frac{1}{4} = f_{XY}(x, y) = \frac{1}{2} \cdot \frac{1}{2} = f_X(x) \cdot f_Y(y)$$

0 otherwise if both x & y are in $[-1, 1]$

X and Y are uniformly distributed in
the disc $x^2+y^2\leq 1$



Are they independent?

A. yes

B. no

C. I could not figure it out

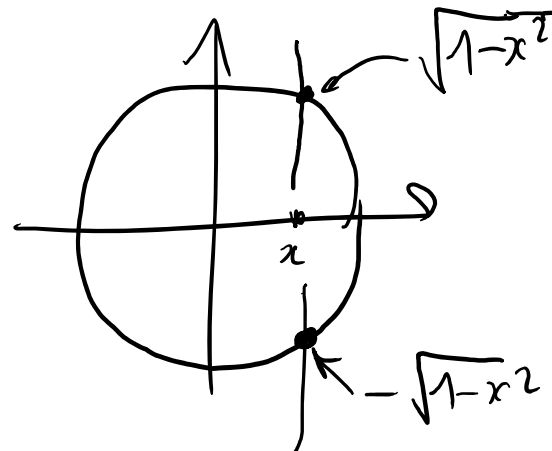
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Joint PDF $f_{XY}(x, y) = \frac{1}{\text{area}} = \frac{1}{\pi}$ if x, y in the disc

Marginal distributions: 0 - otherwise

$$f_X(x) = \int_{-\infty}^{+\infty} f_{XY}(x, y) dy = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{dy}{\pi} = \frac{2\sqrt{1-x^2}}{\pi}$$

Same for $f_Y(y) = \frac{2\sqrt{1-y^2}}{\pi}$



$$\frac{1}{\pi} = f_{XY}(x, y) \neq \frac{2}{\pi} \sqrt{1-x^2} \cdot \frac{2}{\pi} \sqrt{1-y^2} = f_X(x) \cdot f_Y(y)$$

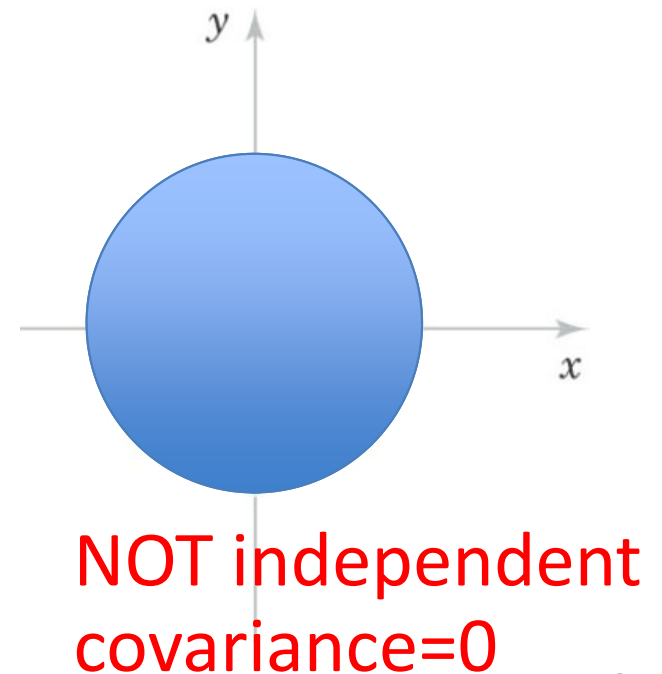
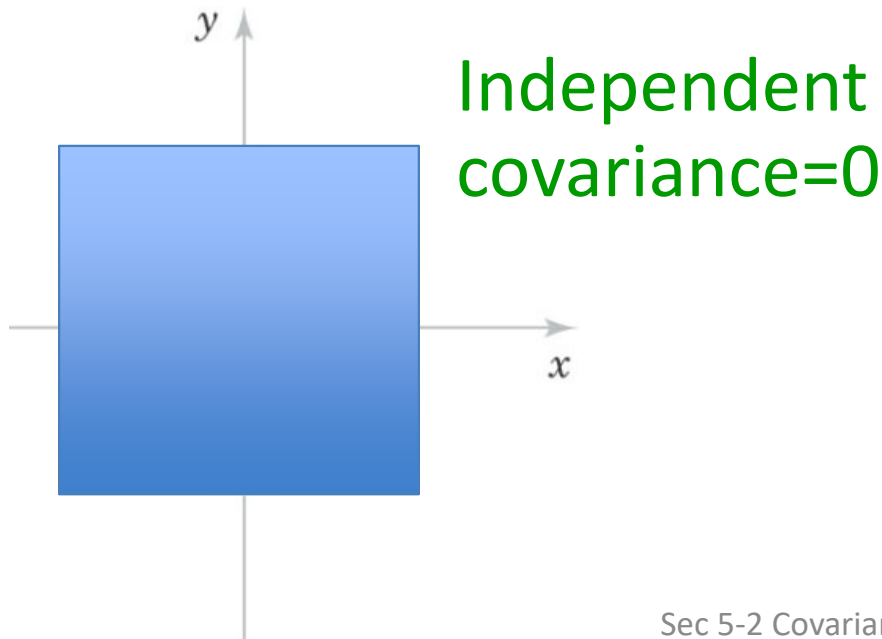
Variables are NOT independent

Independence Implies $\sigma = \rho = 0$ but not vice versa

- If X and Y are independent random variables,

$$\sigma_{XY} = \rho_{XY} = 0 \quad (5-17)$$

- $\rho_{XY} = 0$ is necessary, but **not a sufficient** condition for independence.



Correlation is “normalized covariance”

- Also called:
Pearson correlation coefficient

$$\rho_{XY} = \sigma_{XY} / \sigma_X \sigma_Y$$

is the covariance
normalized to
be $-1 \leq \rho_{XY} \leq 1$



Karl Pearson (1852– 1936)

English mathematician and biostatistician

Prove that ρ_{xy} is in $[-1, 1]$

$$Z_x = \frac{X - \mu_x}{\sigma_x}; \quad Z_y = \frac{Y - \mu_y}{\sigma_y}$$

$$0 \leq E((Z_x - Z_y)^2) = E(Z_x^2) + E(Z_y^2) - 2E(Z_x \cdot Z_y) = 2 - 2 \frac{1}{\sigma_x \sigma_y} E((X - \mu_x)(Y - \mu_y)) =$$

$$2 - 2\rho_{xy} \implies \boxed{\rho_{xy} \leq 1}$$

$$0 \leq E((Z_x + Z_y)^2) = E(Z_x^2) + E(Z_y^2) + 2E(Z_x \cdot Z_y) = 2 + 2\rho_{xy} \implies$$

$$\implies \boxed{\rho_{xy} \geq -1}$$

Spearman rank correlation

- **Pearson correlation** tests for **linear relationship** between X and Y
- **Unlikely for** variables with **broad distributions** → non-linear effects dominate
- **Spearman correlation** tests for any **monotonic relationship** between X and Y
- **Calculate ranks** (1 to n), $r_X(i)$ and $r_Y(i)$ of variables in both samples. Calculate Pearson correlation between ranks:
 $Spearman(X,Y) = Pearson(r_X, r_Y)$
- **Ties:** convert to fractions, e.g. tie for 6s and 7s place both get 6.5. This can lead to artefacts.
- If lots of ties: use **Kendall rank correlation** (Kendall tau)

Credit: XKCD
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Matlab exercise: Correlation/Covariation

- Generate a sample with **Stats=100,000** of two Gaussian random variables **r1** and **r2** which have **mean 0** and **standard deviation 2** and are:
 - **Uncorrelated**
 - Correlated with **correlation coefficient 0.9**
 - Correlated with **correlation coefficient -0.5**
 - Trick: first make **uncorrelated r1** and **r2**. Then make a new variable: **$r1_{mix} = mix \cdot r2 + (1 - mix^2)^{0.5} \cdot r1$** ; where **mix = corr. coeff.**
- For each value of **mix** calculate covariance and **correlation coefficient** between **r1mix** and **r2**
- In each case make a scatter plot: **`plot(r1mix,r2,'k.')`**;

Matlab exercise: Correlation/Covariation

1. Stats=100000;
2. r1=2.*randn(Stats,1);
3. r2=2.*randn(Stats,1);
4. disp('Covariance matrix='); disp(cov(r1,r2));
5. disp('Correlation=');disp(corr(r1,r2));
6. figure; plot(r1,r2,'k.');
7. mix=0.9; **%Mixes r2 to r1 but keeps same variance**
8. r1mix=mix.*r2+sqrt(1-mix.^2).*r1;
9. disp('Covariance matrix='); disp(cov(r1mix,r2));
10. disp('Correlation=');disp(corr(r1mix,r2));
11. figure; plot(r1mix,r2,'k.');
12. mix=-0.5; **%REDO LINES 8-11**