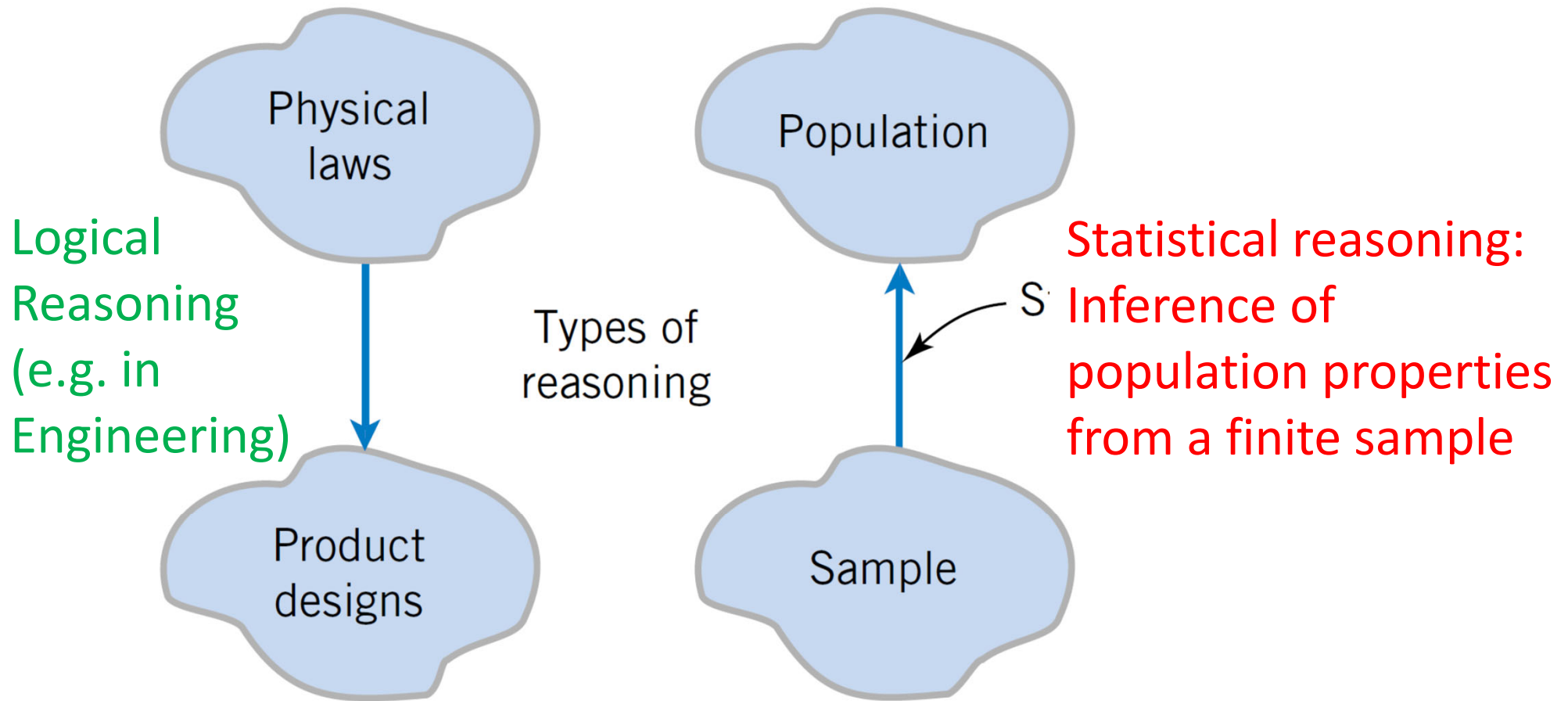


This Thursday 4/3/2025  
we will have a remedial Matlab lab.  
Bring your laptops

Same time (9:30am-10:50am)  
and place (here)  
as our usual lecture

Descriptive statistics:  
Populations, Samples  
Histograms, Quartiles  
Sample mean and  
variance

# Two types of reasoning



# Numerical Summaries of Data

- Data are the **numerical observations** of a **phenomenon of interest**.
- The totality of all observations is a **population**.
  - **Population can be infinite** (e.g. abstract random variables)
  - **It can be very large** (e.g. 7 billion humans or all patients who have cancer of a given type)
- A (usually small) portion of the population collected for analysis is a random **sample**.
- We want to **use sample** to **infer facts about populations**
- The **inference** is not perfect but **gets better and better as sample size increases**.

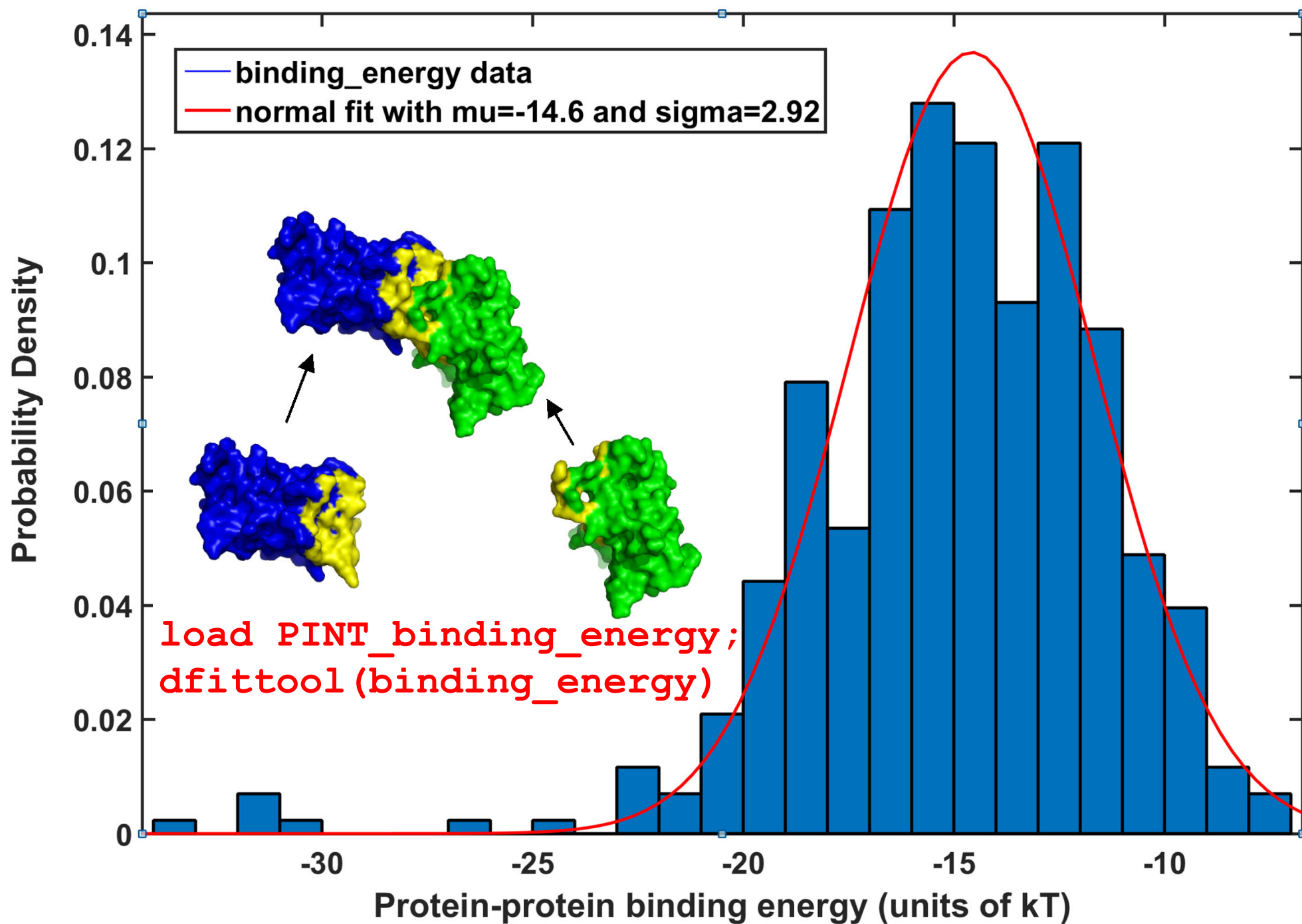
# Some Definitions

- The random variables  $X_1, X_2, \dots, X_n$  are a **random sample** of **size  $n$**  if:
  - a) The  $X_i$  are **independent** random variables.
  - b) Every  $X_i$  has **the same probability distribution**.
- Such  $X_1, X_2, \dots, X_n$  are also called **independent and identically distributed** (or **i. i. d.**) random variables

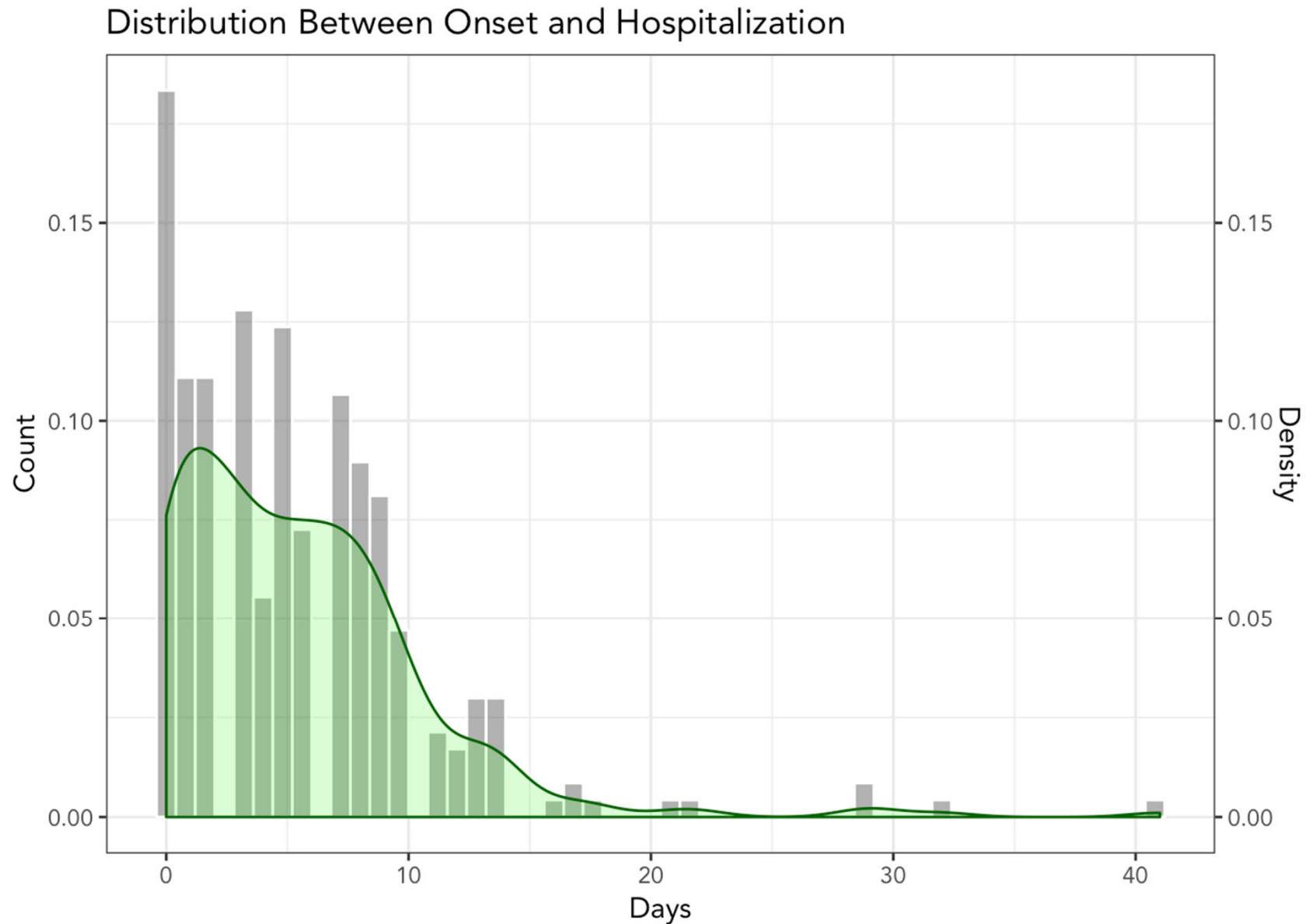
Ways to describe a sample:

**Histogram**

approximates PDF  
(or PMF)



# PDF of time between COVID-19 symptoms onset and hospitalization in IL, April 2020



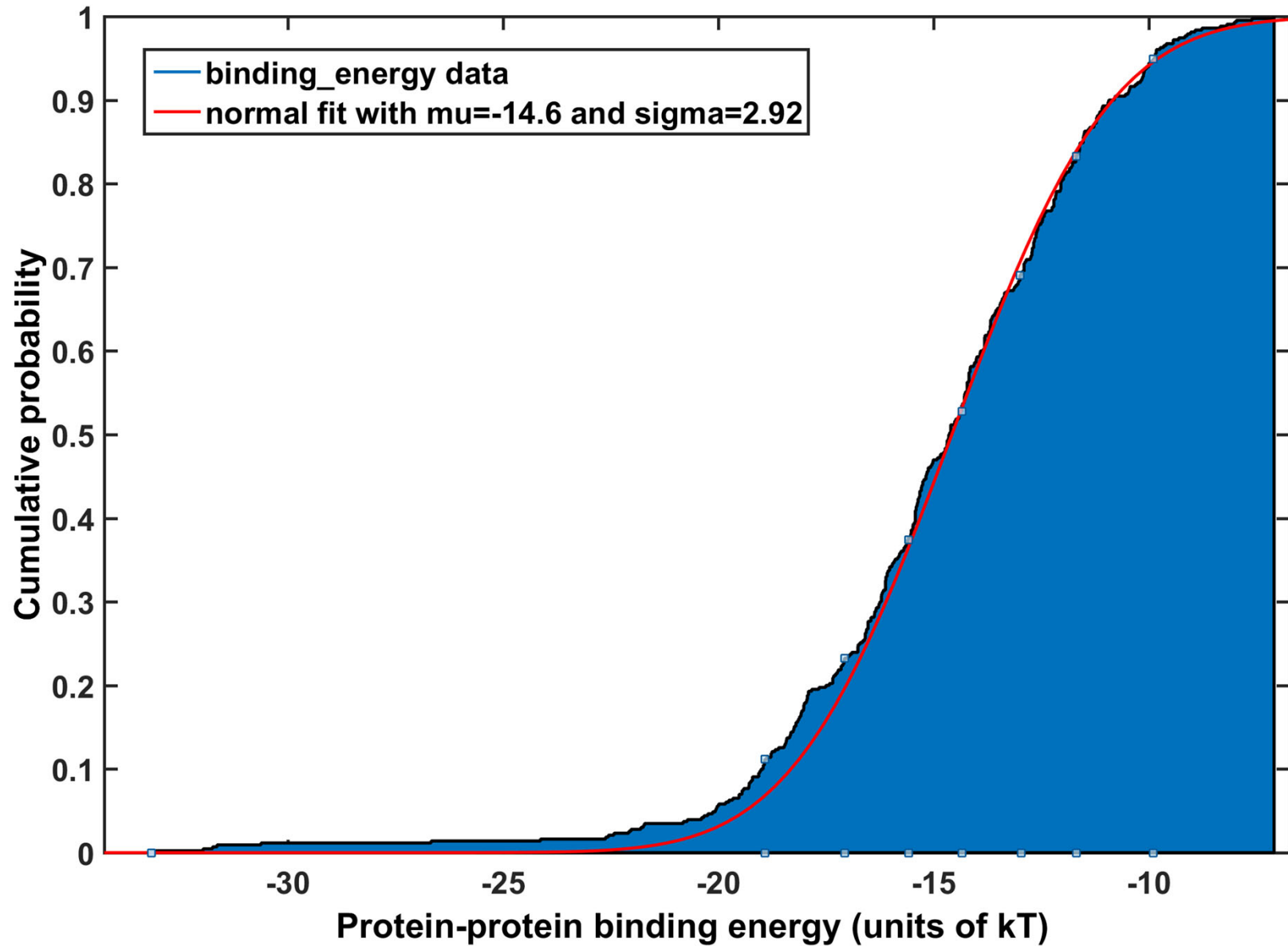


# Histograms with Unequal Bin Widths

- If the data is tightly clustered in some regions and scattered in others, it is visually helpful to use **narrow bin widths** in the **clustered region** and **wide bin widths** in the **scattered areas**.
- To approximate the PDF, the **rectangle area**, not the height, must be proportional to the **bin relative frequency**.

$$\text{Rectangle height} = \frac{\text{bin relative frequency}}{\text{bin width}}$$

# Cumulative Frequency Plot



# Median, Quartiles, Percentiles

- The **median**  $q_2$  divides the sample into two equal parts: 50% ( $n/2$ ) of sample points below  $q_2$  and 50% ( $n/2$ ) points above  $q_2$
- The **three quartiles** partition the data into four equally sized counts or segments.
  - 25% of the data is less than  $q_1$ .
  - 50% of the data is less than  $q_2$ , the median.
  - 75% of the data is less than  $q_3$ .
- There are **100 percentiles**.  $n$ -th percentile  $p_n$  is defined so that  $n\%$  of the data is less than  $p_n$

# Box-and-Whisker Plot

- A box plot is a graphical display showing **S**pread, **O**utliers, **C**enter, and **S**hape (**SOCS**).
- It displays the **5-number summary**: *min*,  $q_1$ , *median*,  $q_3$ , and *max*.

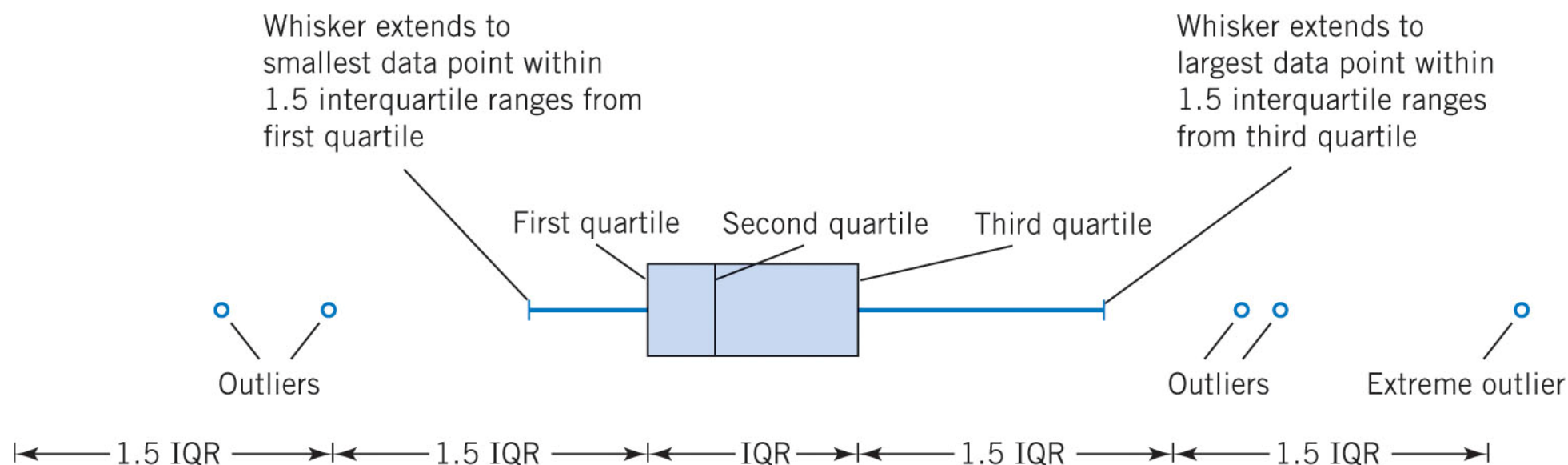


Figure 6-13 Description of a box plot.



Credit: XKCD  
comics

WHY ARE THERE SLAVES IN THE BIBLE

WHY DO TWINS HAVE DIFFERENT FINGERPRINTS  
WHY ARE AMERICANS AFRAID OF DRAGONS

# QUESTIONS

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WHY IS STEALING WRONG

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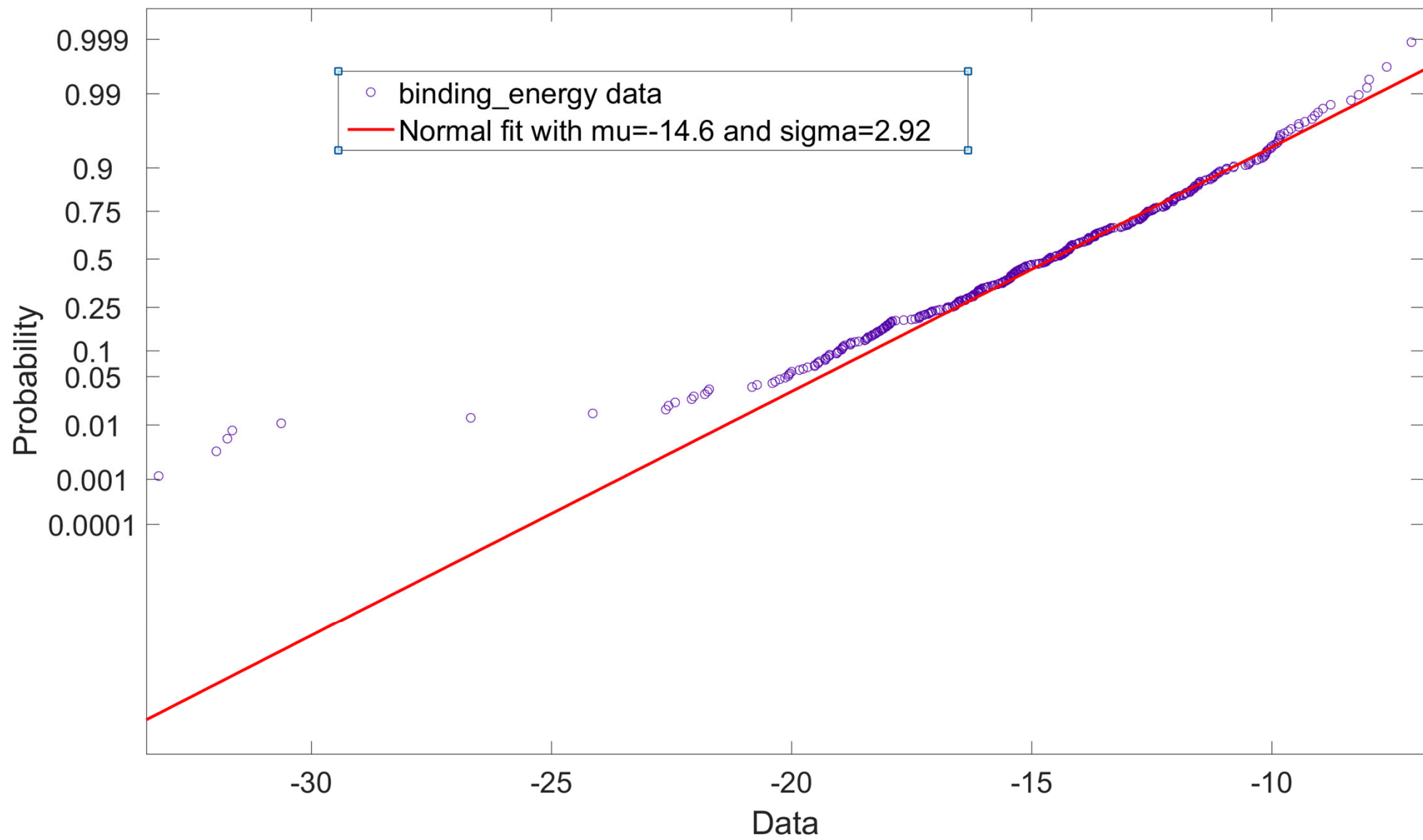


WHY IS SEX  
SO IMPORTANT



# Probability Plots

- How do we know if a particular probability distribution is a reasonable model for a data set?
- A histogram of a large data set reveals the shape of a distribution. The histogram of a small data set does not provide a clear picture.
- A probability plot is helpful for all data set size. How good is the model based on a particular probability distribution can be verified using a subjective visual examination.

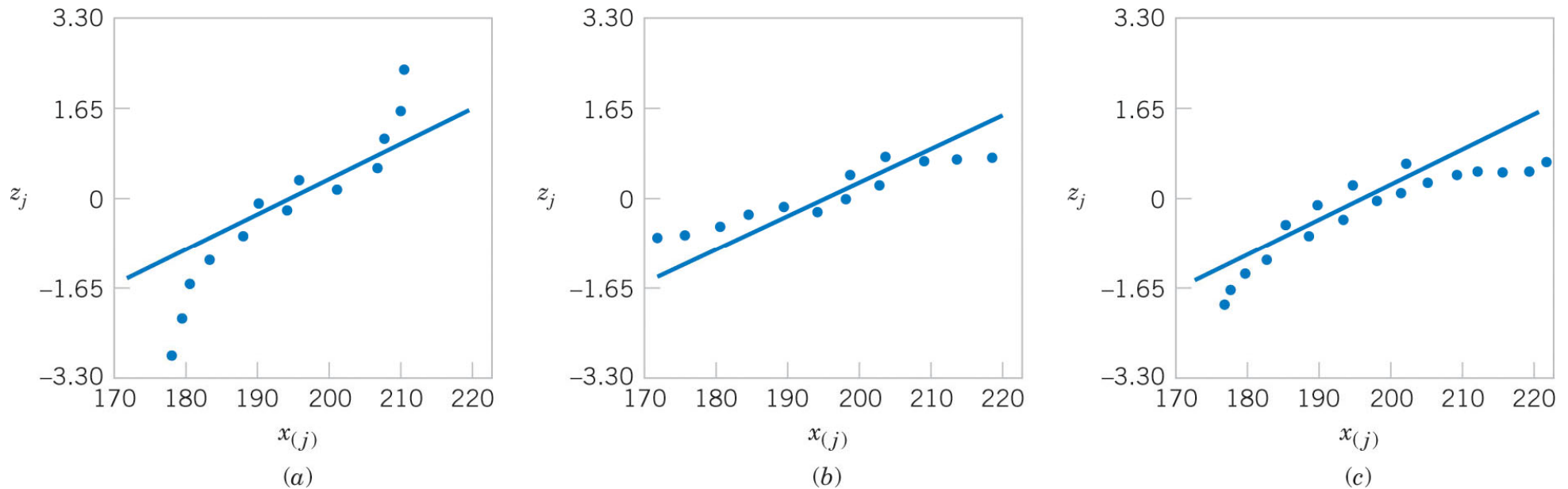


# How To Build a Probability Plot

- Sort the data observations in ascending order:  
 $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ .
- Empirically determined cumulative frequency  
 $\text{Prob}(x \leq x_{(j)}) = j/n$ . To correct for discreteness of  $x_{(j)}$  better use  $\text{Prob}(x \leq x_{(j)}) = (j-0.5)/n$
- If you believe that  $\text{CDF}(x)$  describes your random variable  $(j-0.5)/n$  should be close to  $\text{CDF}(x_{(j)})$
- Probability plot is  $x_{(j)} \cdot [(j-0.5)/n] / \text{CDF}(x_{(j)})$  plotted versus the observed value  $x_{(j)}$ .
- If the fit is good - one gets a straight line
- Deviations can be seen especially at tails.



# Probability Plot Variations



**Figure 6-21** Normal probability plots indicating a non-normal distribution.

(a) Light tailed distribution (squeezed together)

(b) Heavy tailed distribution (stretched out)

(c) Right skewed distribution (left end squeezed, right end stretched)

Credit: XKCD  
comics

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WHY IS PSYCHIC WEAK TO BUG

WHY DO CHILDREN GET CANCER

WHY IS POSEIDON ANGRY WITH ODYSSEUS

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# Linear Functions of Random Variables

- A **function of multiple random variables** is itself **a random variable**.
- A function of random variables can be formed by either **linear** or **nonlinear relationships**. We will only work with linear functions.
- Given random variables  $X_1, X_2, \dots, X_p$  and constants  $c_1, c_2, \dots, c_p$   
$$Y = c_1X_1 + c_2X_2 + \dots + c_pX_p \quad (5-24)$$
is a linear combination of  $X_1, X_2, \dots, X_p$ .



# Mean & Variance of a Linear Function

$$Y = c_1X_1 + c_2X_2 + \dots + c_pX_p$$

$$E(Y) = c_1E(X_1) + c_2E(X_2) + \dots + c_pE(X_p) \quad (5-25)$$

$$V(Y) = c_1^2V(X_1) + c_2^2V(X_2) + \dots + c_p^2V(X_p) + 2 \sum_{i < j} \sum c_i c_j \text{cov}(X_i X_j) \quad (5-26)$$

If  $X_1, X_2, \dots, X_p$  are **independent**, then  $\text{cov}(X_i X_j) = 0$ ,

$$V(Y) = c_1^2V(X_1) + c_2^2V(X_2) + \dots + c_p^2V(X_p) \quad (5-27)$$

Descriptive statistics:  
Sample mean and  
variance



# Some Definitions

- The random variables  $X_1, X_2, \dots, X_n$  are a **random sample** of **size  $n$**  if:
  - a) The  $X_i$  are **independent** random variables.
  - b) Every  $X_i$  has **the same probability distribution**.

Such  $X_1, X_2, \dots, X_n$  are also called independent and identically distributed (or **i. i. d.**) random variables

- A **statistic** is any function of the observations in a random sample.
- The probability distribution of a statistic is called a **sampling distribution**.





# Statistic #1: Sample Mean

If the  $n$  observations in a random sample are denoted by  $x_1, x_2, \dots, x_n$ , the **sample mean** is

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum_{i=1}^n x_i}{n} \quad (6-1)$$

## IMPORTANT:

Sample mean  $\bar{X}$  is drawn from a random variable

$$\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n}$$

$$E(\bar{X}) = \frac{n \cdot E(X_i)}{n} = \frac{n \cdot \mu}{n} = \mu$$

$$V(\bar{X}) = \frac{n \cdot V(X_i)}{n^2} = \frac{n \cdot \sigma^2}{n} = \frac{\sigma^2}{n}$$

$$\text{Stand. dev. } (\bar{X}) = \frac{\sigma}{\sqrt{n}}$$

# Mean & Variance of the Sample Average

$$\text{If } \bar{X} = \frac{(X_1 + X_2 + \dots + X_n)}{n} \text{ and } E(X_i) = \mu$$

$$\text{Then } E(\bar{X}) = \frac{n \cdot \mu}{n} = \mu \quad (5-28a)$$

If the  $X_i$  are independent with  $V(X_i) = \sigma^2$

$$\text{Then } V(\bar{X}) = \frac{n \cdot \sigma^2}{n^2} = \frac{\sigma^2}{n} \quad (5-28b)$$

# Central Limit Theorem

If  $X_1, X_2, \dots, X_n$  is a random sample of size  $n$  is taken from a population with mean  $\mu$  and **finite variance  $\sigma^2$** , and **any distribution**. If  $\bar{X}$  is the sample mean, then the **limiting form of the distribution** of

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad (7-1)$$

for **large  $n$** , is the **standard normal distribution**.

If  $X_1, X_2, \dots, X_n$  are themselves normally distributed – for any  $n$

# Test CLT for your own random variable

- Go to:  
[https://onlinestatbook.com/stat\\_sim/sampling\\_dist/](https://onlinestatbook.com/stat_sim/sampling_dist/)
- Select “Custom” at the top and use mouse to sketch the PMF of your own random variable
- Select “mean” and  $n=5$  in the third panel
- Choose “Animated” in the second panel and use number\_of\_experiments=5 to see one sample being generated
- Repeat with number\_of\_experiments =10,000
- Now select “mean” and  $n=25$  in the fourth panel
- Skewness and Curtosis are measures of how good is the normal (Gaussian) fit (choose “fit normal”)

# Sampling Distributions of Sample Means

**Figure 7-1** Distributions of average scores from throwing dice.

Mean =  $(6+1)/2=3.5$

Sigma<sup>2</sup> =  $[(6-1+1)^2-1]/12=2.92$

Sigma=1.71

## Formulas

$$\mu = \frac{b + a}{2}$$

$$\sigma_X^2 = \frac{(b - a + 1)^2 - 1}{12}$$

$$\sigma_{\bar{X}}^2 = \frac{\sigma_X^2}{n}$$

show  
Matlab

