

Midterm Info

- Closed book exam at CBTF; no books, notes, laptops, phones...
- Calculators (not on smartphones) are provided by CBTF
- The following two printouts and lecture slides 1-13 part 1 and part 2 will be provided

Name	Probability Distribution	Mean	Variance	Section in Book
Discrete				
Uniform	$\frac{1}{n}, a \leq b$	$\frac{(b + a)}{2}$	$\frac{(b - a + 1)^2 - 1}{12}$	3-5
Binomial	$\binom{n}{x} p^x (1 - p)^{n-x},$ $x = 0, 1, \dots, n, 0 \leq p \leq 1$	np	$np(1 - p)$	3-6
Geometric	$(1 - p)^{x-1} p,$ $x = 1, 2, \dots, 0 \leq p \leq 1$	$1/p$	$(1 - p)/p^2$	3-7.1
Negative binomial	$\binom{x-1}{r-1} (1 - p)^{x-r} p^r$ $x = r, r + 1, r + 2, \dots, 0 \leq p \leq 1$	r/p	$r(1 - p)/p^2$	3-7.2
Hypergeometric	$\frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}$ $x = \max(0, n - N + K), 1, \dots$ $\min(K, n), K \leq N, n \leq N$	$np,$ where $p = \frac{K}{N}$	$np(1 - p) \left(\frac{N-n}{N-1} \right)$	3-8
Poisson	$\frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2, \dots, 0 < \lambda$	λ	λ	3-9
Continuous				
Uniform	$\frac{1}{b - a}, a \leq x \leq b$	$\frac{(b + a)}{2}$	$\frac{(b - a)^2}{12}$	4-5
Normal	$\frac{1}{\sigma \sqrt{2\pi}} e^{-1/2 \left(\frac{x-\mu}{\sigma} \right)^2}$ $-\infty < x < \infty, -\infty < \mu < \infty, 0 < \sigma$	μ	σ^2	4-6
Exponential	$\lambda e^{-\lambda x}, 0 \leq x, 0 < \lambda$	$1/\lambda$	$1/\lambda^2$	4-8
Erlang	$\frac{\lambda^r x^{r-1} e^{-\lambda x}}{(r - 1)!}, 0 < x, r = 1, 2, \dots$	r/λ	r/λ^2	4-9.1
Gamma	$\frac{\lambda^r x^{r-1} e^{-\lambda x}}{\Gamma(r)}, 0 < x, 0 < r, 0 < \lambda$	r/λ	r/λ^2	4-9.2

What is included in the midterm?

- Probability of events (set operations)
- Multiplication rules. Combinatorics
- Bayes Theorem
- Discrete Random Variables
- See HW1 for inspiration
- No Matlab exercises (since no access to Matlab at CBTF)

Probability Multiplication Rules

Combinatorics

Mr. Jones has 6 different books that he is going to put on his bookshelf. Of these, 3 are chemistry books, 2 are physics books, and 1 is a mathematics book. Jones wants to arrange his books so that two conditions are met:

(1) all the books dealing with the same subject are together on the shelf

AND

(2) all chemistry books are on the leftmost side.

How many such different arrangements are possible?

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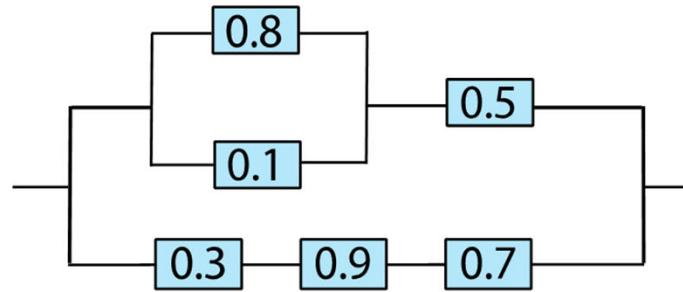
AND

(2) all chemistry books are on the leftmost side.

How many such different arrangements are possible?

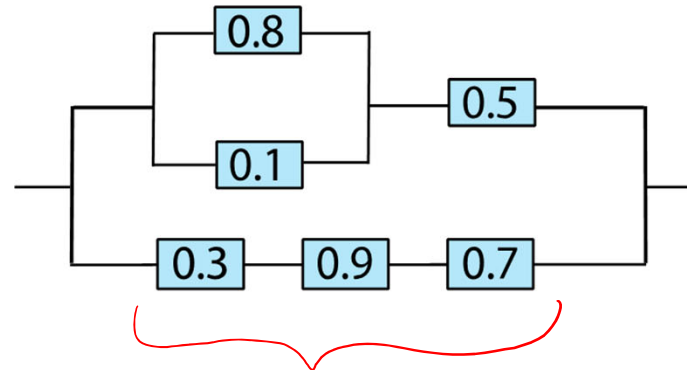
Answer: $(3! * 2! * 1!) * 2! = 24$

4. (4 points) The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



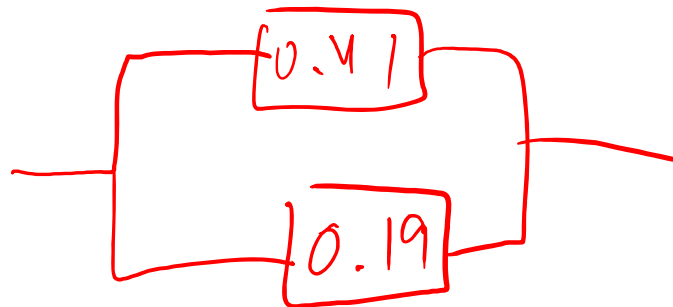
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$$1 - (1 - 0.8) \cdot (1 - 0.1) = 0.82$$



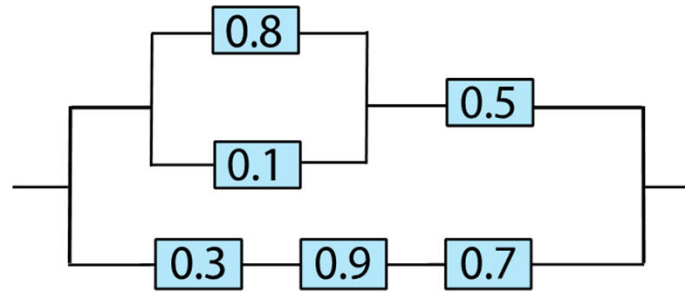
$$0.3 \cdot 0.9 \cdot 0.7 = 0.19$$

$$0.82 \times 0.5 = 0.41$$



$$1 - (1 - 0.41) \cdot (1 - 0.19) = 0.52$$

4. (4 points) The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



Answer: $P(\text{Operate}) = 1 - (1 - 0.3 * 0.9 * 0.7) * (1 - 0.5 * (1 - (1 - 0.8) * (1 - 0.1))) = \underline{0.52}$

Bayes theorem

In answering a question on a multiple-choice test, a student either knows the answer or he guesses. Let $\frac{1}{3}$ be the probability that he knows the answer. If he does not know the answer, he randomly guesses one out of 4 multiple choice questions. What is the conditional probability that a student knew the answer to a question given that he answered it correctly?

- A. $\frac{1}{4}$
- B. $\frac{1}{3}$
- C. $\frac{2}{3}$
- D. $\frac{1}{5}$
- E. I don't know

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Answer: $P(K)=1/3$, $P(K')=2/3$, $P(C|K)=1$, $P(C|K')=1/4$.

$P(K|C)=P(C|K)*P(K)/P(C)=1*(1/3)/(1*1/3+(1/4)*(2/3))=2/3=0.666\dots$

(10 points) Suppose that a bag contains ten coins, three of which are fair, while the remaining seven are biased: they have probability of 0.6 of heads when flipped. A coin was taken at random from the bag and flipped five times. All five flips gave heads. What's the probability that this coin is fair?

(10 points) Suppose that a bag contains ten coins, three of which are fair, the remaining seven having probability 0.6 of giving heads when flipped. A coin is taken at random from the bag and flipped five times. All five flips give heads. What's the probability that a coin is fair given the five coin flips?

Answer: Let us denote H_1 as the hypothesis that a coin is fair and H_2 that a coin is biased. The data that all five flips were heads is denoted as D . Therefore,

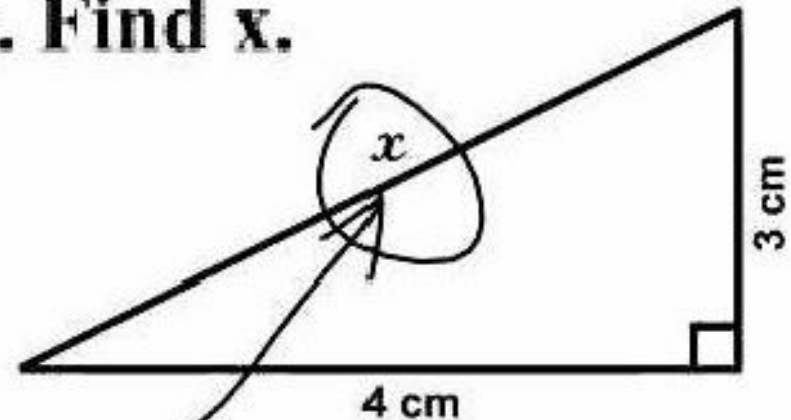
$$P(H_1|D) = P(D|H_1)P(H_1)/P(D) = 0.5^5 * 0.3 / (0.5^5 * 0.3 + 0.6^5 * 0.7) = 0.147$$

Discrete Probability Distributions

What is X in this problem?

- What is the random variable: Look for keywords:
 - Find the probability that....
 - What is the mean (or variance) of...
- What are parameters? Look for keywords:
 - Given that...
 - Assuming that...

3. Find x.



Here it is

Guide to probability distributions

- Binomial: # of samples, n , is fixed, # of successes, x , is variable

$$P(X=x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

- Geometric: # of samples, x is variable. # of successes 1 is fixed. Success comes in the end

$$P(X=x) = (1-p)^{x-1} \cdot p$$

- Negative binomial: # of samples, x is variable. # of successes, r , is fixed r th success in the end

$$P(X=x) = \frac{(x-1)!}{(r-1)!(x-r)!} p^r (1-p)^{x-r}$$

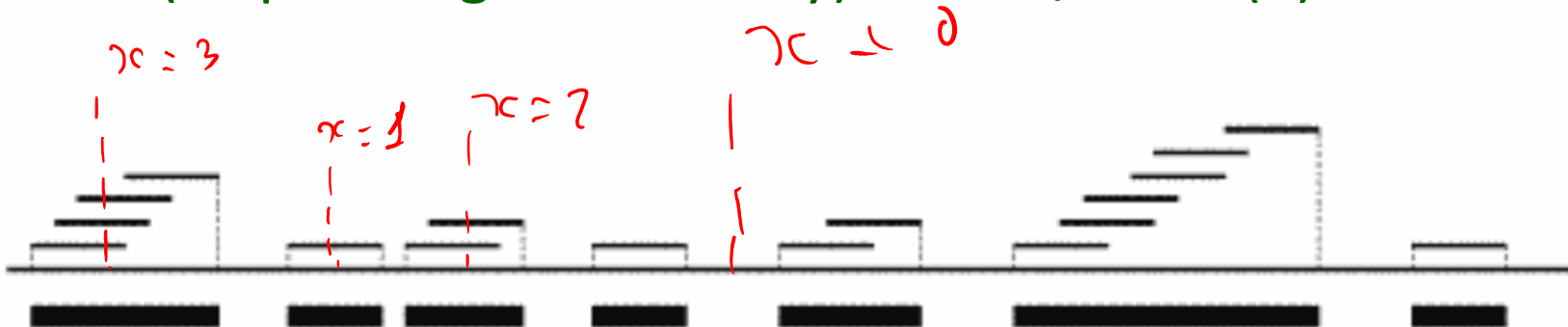
Poisson distribution in genomics

- G - genome length (in bp)
- L - short read average length
- N - number of short read sequenced
- λ - sequencing redundancy = LN/G
- x - number of short reads covering a given site on the genome

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Ewens, Grant, Chapter 5.1

Poisson as a limit of Binomial. For a given site on the genome for each short read Prob(site covered): $p=L/G$ is very small. Number of attempts (short reads): N is very large. Their product (sequencing redundancy): $\lambda = NL/G$ is $O(1)$.



Probability that a base pair in the genome is not covered by any short reads is 0.1

One randomly selects base pairs until exactly 5 uncovered base pairs are found.

Which discrete probability distribution describes the number of attempts?

- A. Poisson
- B. Binomial
- C. Geometric
- D. Negative Binomial
- E. I have no idea

Poisson	$\frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2, \dots, 0 < \lambda$
Binomial	$\binom{n}{x} p^x (1-p)^{n-x}$ $x = 0, 1, \dots, n, 0 \leq p \leq 1$
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- B. $p=0.1, r=0.5$
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Cancer happens when the gene p53 mutates.

Probability of p53 to mutate per year is 5%.

How many years before a patient gets disease?

Which discrete probability distribution
would you use to answer?

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- B. Binomial
- C. Geometric
- D. Negative Binomial
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