BIOE 210

PRACTICE EXAM 3

You have 80 minutes to complete this exam.
You may use notes or printouts from the course website,
but **no electronic resources**.

Circle your final answer for each question.

Part I (20 points; 5 points each)

(1) True or False. If a nonlinear system has only one solution, Newton's method will converge from any initial guess.

(2) A genetic counselor informs a patient that their odds of getting disease are 1 in 300. What is the probability the patient will get the disease?

(3) True or False. Every square matrix has a pseudoinverse.

(4) True or False. It is necessary to have more observations than parameters to find a least-squares estimate using linear regression.

Part II (40 points)

You are given a set of inputs ($\mathbf{x}$) and responses ($\mathbf{y}$)

$\mathbf{x}$	$\mathbf{y}$
0	2.1
1	3.1
2	5.8
3	11.3

You hypothesize that these data fit one of two models: $y = \beta_0 + \beta_1 x + \beta_2 x^2$ or $y = \beta_0 + \beta_1 e^x$. You want to fit the parameters using linear regression.

Construct a design matrix for each model using the data in the above table. (The design matrix is the matrix $\mathbf{X}$ in the linear system $\mathbf{y} = \mathbf{X}\beta + \epsilon$.)

You use `fitlm` to fit both models, with the following outputs.

```
Linear regression model:
    y ~ 1 + x + x^2

Estimated Coefficients:
                   Estimate      SE        tStat      pValue  
                   --------    -------    -------    --------

    (Intercept)     2.155      0.23974     8.9889    0.070533
    x              -0.345        0.385    -0.8961     0.53485
    x^2             1.125      0.12298     9.1476    0.069319


Number of observations: 4, Error degrees of freedom: 1
Root Mean Squared Error: 0.246
R-squared: 0.999,  Adjusted R-Squared 0.996
F-statistic vs. constant model: 421, p-value = 0.0344



Linear regression model:
    y ~ 1 + expx

Estimated Coefficients:
                   Estimate       SE        tStat      pValue  
                   --------    --------    ------    ---------

    (Intercept)     1.8553      0.25642    7.2353     0.018572
    expx           0.47699     0.023746    20.087    0.0024692


Number of observations: 4, Error degrees of freedom: 2
Root Mean Squared Error: 0.355
R-squared: 0.995,  Adjusted R-Squared 0.993
F-statistic vs. constant model: 403, p-value = 0.00247
```

Which model would give better predictions? Why?

Part III (40 points)

Find the Jacobian matrix for the following system of equations:

$$\mathbf{f}(\mathbf{x}) = \begin{pmatrix} x_2 - 3x_1 \\ 2(x_1 + x_2) \end{pmatrix}$$

Starting with an initial guess of $\mathbf{x}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, find $\mathbf{x}_1$. Use $\mathbf{J}(\mathbf{x}_0)^{-1} = \begin{pmatrix} -1/4 & 1/8 \\ 1/4 & 3/8 \end{pmatrix}$.

Evaluate $\mathbf{f}(\mathbf{x}_1)$ to show that $\mathbf{x}_1$ is a root of $\mathbf{f}$.

Why would Newton's method converge to a root after only one iteration?