BIOE 210, SPRING 2019

PRACTICE EXAM 2

You have 80 minutes to complete this exam.
You may use notes or printouts from the course website,
but **no electronic resources**.

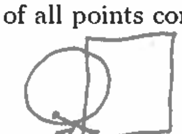
Circle your final answer for each question.

PART I (18 POINTS; 3 POINTS EACH)

(1) What is the distance between the origin and the hyperplane $3x_1 + 2x_2 - x_3 = 0$?

$$d = \frac{0}{\|\underline{a}\|} = \boxed{0}$$

(2) TRUE or FALSE. The union of any two convex sets is also convex. (Note: the union of two sets is the set of all points contained in either set.)



(3) Is the vector $\begin{pmatrix} a \\ b \end{pmatrix}$ an eigenvector of the matrix $\begin{pmatrix} 3 & 4/b \\ 0 & 1 \end{pmatrix}$?

$$\begin{pmatrix} 3 & 4/b \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3a+4 \\ b \end{pmatrix} \neq \lambda \begin{pmatrix} a \\ b \end{pmatrix}$$

No, it is not.

~~(4) TRUE or FALSE. Let $A \in \mathbb{R}^{n\times n}$. The matrix A is perfect if the SVD of A reveals n distinct singular values (the diagonal entries along Σ).~~

MATRIX Decompositions are not on this exam.

(5) We showed that quadratic programs can be solved to global optimality if the quadratic objective is convex. Is the quadratic program that results from the Support Vector Machine (SVM) problem solvable to global optimality? Why or why not?

Yes. $\max \frac{2}{\|\underline{a}\|} = \min \sum_i a_i^2$ is convex

(pure quadratic with nonnegative coefficients)

(6) Give a set of three vectors that span the space $\mathbb{R}^2$.

$$\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$$

can be anything

Can your set of vectors also serve as a basis for $\mathbb{R}^2$?

No. No 3 vectors can be a basis for $\mathbb{R}^2$.

Part II (30 points)

Construct an orthonormal basis from the vectors $\left\{\begin{pmatrix}3\\4\end{pmatrix}, \begin{pmatrix}0\\-1\end{pmatrix}\right\}$.

$$u_1 = \begin{pmatrix}3\\4\end{pmatrix}$$

$$u_2 = \begin{pmatrix}0\\-1\end{pmatrix} - \underbrace{\text{proj}_{u_1}\begin{pmatrix}0\\-1\end{pmatrix}}_{\frac{\begin{pmatrix}0\\-1\end{pmatrix}\cdot\begin{pmatrix}3\\4\end{pmatrix}}{\begin{pmatrix}3\\4\end{pmatrix}\cdot\begin{pmatrix}3\\4\end{pmatrix}}\begin{pmatrix}3\\4\end{pmatrix} = \frac{-4}{25}\begin{pmatrix}3\\4\end{pmatrix}}$$

$$= \begin{pmatrix}0\\-1\end{pmatrix} + \frac{4}{25}\begin{pmatrix}3\\4\end{pmatrix} = \frac{25}{25}\begin{pmatrix}0\\-1\end{pmatrix} + \frac{4}{25}\begin{pmatrix}3\\4\end{pmatrix}$$

$$= \begin{pmatrix}12/25\\-9/25\end{pmatrix}$$

$$\|u_2\| = \sqrt{\left(\frac{12}{25}\right)^2 + \left(\frac{-9}{25}\right)^2}$$

$$= \frac{1}{25}\sqrt{12^2+9^2}$$

$$= 15/25 = 3/5$$

$$\boxed{\hat{u}_1 = \begin{pmatrix}3/5\\4/5\end{pmatrix},\ \hat{u}_2 = \begin{pmatrix}4/5\\-3/5\end{pmatrix}}$$

Decompose the vector $\begin{pmatrix}1\\-1\end{pmatrix}$ onto your basis.

$$a_1 = \begin{pmatrix}1\\-1\end{pmatrix}\cdot\begin{pmatrix}3/5\\4/5\end{pmatrix} = -\frac{1}{5}$$

$$a_2 = \begin{pmatrix}1\\-1\end{pmatrix}\cdot\begin{pmatrix}4/5\\-3/5\end{pmatrix} = \frac{7}{5}$$

$$\boxed{\begin{pmatrix}1\\-1\end{pmatrix} = -\frac{1}{5}\begin{pmatrix}3/5\\4/5\end{pmatrix} + \frac{7}{5}\begin{pmatrix}4/5\\-3/5\end{pmatrix}}$$

Part IV (30 points)

Find the intersection of the planes defined by

$$
\begin{aligned}
3x - 4y + 2z &= 10 \\
2x - z &= -2 \\
-x - 4y + 4z &= 14
\end{aligned}
$$

$$
\begin{pmatrix} 3 & -4 & 2 & 10 \\ 2 & 0 & -1 & -2 \\ -1 & -4 & 4 & 14 \end{pmatrix}
\xrightarrow{R_1 \Leftrightarrow R_2}
\xrightarrow{\frac{1}{2}R_1}
\begin{pmatrix} 1 & 0 & -1/2 & -1 \\ 3 & -4 & 2 & 10 \\ -1 & -4 & 4 & 14 \end{pmatrix}
$$

$$
\xrightarrow{R_2 - 3R_1}
\xrightarrow{R_3 + R_1}
\begin{pmatrix} 1 & 0 & -1/2 & -1 \\ 0 & -4 & 7/2 & 13 \\ 0 & -4 & 7/2 & 13 \end{pmatrix}
$$

} Rows are the same $\Rightarrow \infty$ sol'ns

$$
\xrightarrow{R_3 - R_2}
\begin{pmatrix} 1 & 0 & -1/2 & -1 \\ 0 & -4 & 7/2 & 13 \\ 0 & 0 & 0 & 0 \end{pmatrix}
$$

Let $x_3 = \alpha$

$$x_2 = \frac{13 - \frac{7}{2}\alpha}{-4}$$

$$x_1 = \frac{1}{2}\alpha - 1$$

We don't find a specific solution using one α; the intersection is the set of all points at all α.

Give a geometric interpretation of the intersection of these planes.

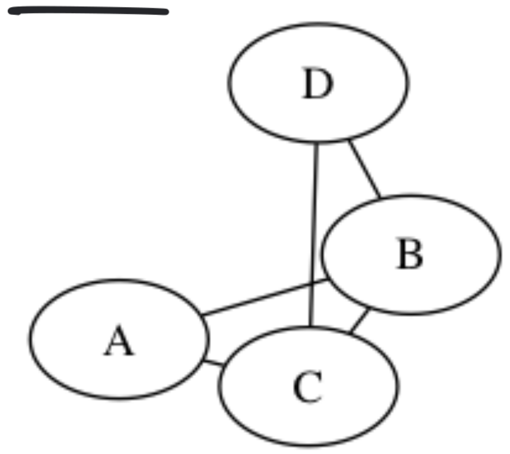
The third plane must contain the line formed by the intersection of the other two

Part IV (20 points)

Define an adjacency matrix **A** for the following four-node, undirected network.

$$A = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{cccc} A & B & C & D \\ \end{array}$$

$$A = \begin{array}{cc} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} & \begin{array}{c} A \\ B \\ C \\ D \end{array} \end{array}$$



~~Give~~ Given the following output from Matlab, report the most "central" node in the network and its centrality score.

```
>> [V,L] = eig(A)

V =

   -0.5573   -0.0000    0.7071   -0.4352
    0.4352    0.7071    0.0000   -0.5573
    0.4352   -0.7071   -0.0000   -0.5573
   -0.5573    0.0000   -0.7071   -0.4352


L =

   -1.5616         0         0         0
         0   -1.0000         0         0
         0         0    0.0000         0
         0         0         0    2.5616
```

largest magnitude

B & C are both equally the most central.

λ max