BIOE 210

PRACTICE EXAM 1

You have 80 minutes to complete this exam.
You may use notes or printouts from the course website,
but **no electronic resources**.

Part I (40 points; 4 points each)

(1) True or False. The matrix $\begin{pmatrix} 3 & 2 & 1 \\ 1 & 0 & -1 \end{pmatrix}$ has an inverse.

(2) True or False. There exists a real number θ such that $\begin{pmatrix} 1 \\ \theta \\ 1/2 \end{pmatrix}$ is a unit vector.

(3) We said (many times) that the integers are not a field since they have additive inverses ($-a$) for every element but not multiplicative inverses (a^{-1}). We can construct a set that contains both additive and multiplicative inverses using the integers by collecting 2^i and -2^i for every integer i:

$$\{\ldots, \pm 2^{-2}, \pm 2^{-1}, \pm 2^0, \pm 2^1, \pm 2^2, \ldots\}$$

Is this set a field?

(4) $\|\mathbf{x}\| = 8$. What is $\|-3\mathbf{x}\|$?

(5) Let $\begin{pmatrix} 0 & 1 & -2 \\ 0 & -1 & 0 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \\ 12 \end{pmatrix}$. What is x_2?

(6) TRUE or FALSE. If the angle between $\mathbf{x} = \begin{pmatrix} 2a \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 4 \\ a \\ 2 \end{pmatrix}$ is 135°, then $\mathbf{x} \cdot \mathbf{y} = 7$.

(7) Which vectors are orthogonal to $\begin{pmatrix} 4 \\ 0 \\ 2 \\ 0 \end{pmatrix}$?

(a) $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1/4 \\ 0 \\ 1/2 \\ 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ (d) $\begin{pmatrix} 0 \\ -12 \\ 0 \\ 8 \end{pmatrix}$

(8) TRUE or FALSE. $\mathbf{AB} \neq \mathbf{BA}$ for all matrices $\mathbf{A}$ and $\mathbf{B}$, even if $\mathbf{A}$ and $\mathbf{B}$ are conformable.

(9) Which of the following differential equations are linear

(a)
$$\frac{\partial^2 u}{\partial x \partial y} + \sin(xy)u = 4$$

(b)
$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial u}{\partial r}\right) = 0$$

(c)
$$\frac{d^2u}{dx^2} + 3e^u x\frac{du}{dx} + u = 1$$

(d)
$$\frac{1}{u}\frac{du}{dt} = t$$

(10) What is the rank of the matrix $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$?

Part II (30 points)

Find the inverse of the matrix $\mathbf{A} = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$

Use the inverse to solve $\mathbf{Ax} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ and $\mathbf{Ax} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

Part III (30 points)

Write equations for the finite difference approximation for the following ODE at **four nodes spanning [0, 3]**.

$$\frac{d^2u}{dx^2} - 4u = x^2, \quad u(0) = 1,\ u(3) = 4$$

Rewrite the equations as a matrix equation of the form $\mathbf{Ax} = \mathbf{y}$.