BIOE 210, SPRING 2019

HOMEWORK 6

Due Wednesday, 5/8/2019 before 9:00am.
Upload a single PDF with your answers to Gradescope.

Part I: Logistic Regression (20 points)

You develop a logistic regression model to predict the chances of developing lung cancer over one's lifetime. The input variables are 1.) the number of cigarettes smoked per day ($\beta = 0.17$), and 2.) the number of cups of coffee consumed per day ($\beta = -0.0003$).

(1) How much do your odds of developing lung cancer increase if you smoke one more cigarette per day?
(2) How much do your odds decrease by drinking another cup of coffee per day?

Part II: Transformation Matrices (40 points)

(1) Using matrix multiplication, rotate the vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ **clockwise** by 60°. Draw the vectors before and after the rotation.
(2) We want to see if rotation and translation commute, i.e. if rotating first followed by translation is the same as translating first and then rotating.
  (a) Compute the matrix product $\mathbf{T}(\Delta x, \Delta y)\mathbf{R}(\theta)$ to find a single matrix that rotates by θ and then translates by Δx and Δy.
  (b) Compute the matrix product $\mathbf{R}(\theta)\mathbf{T}(\Delta x, \Delta y)$ to find a single matrix that translates and then rotates.
  (c) Comparing your two matrices, do the operations commute? Is there any angle θ for which $\mathbf{T}(\Delta x, \Delta y)\mathbf{R}(\theta) = \mathbf{R}(\theta)\mathbf{T}(\Delta x, \Delta y)$? Why or why not?

Part III: Nonlinear Systems (40 points)

We want to find a root for the nonlinear system

$$\mathbf{f}(\mathbf{x}) = \begin{pmatrix} x_2 \cos x_1 \\ 2x_2^2 - 1 \end{pmatrix}$$

We will use multivariate Newton's method to find a vector $\mathbf{x}$ such that $\mathbf{f}(\mathbf{x}) = 0$ starting from an initial guess $\mathbf{x}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(1) Write the Jacobian matrix $\mathbf{J}(\mathbf{x})$ for the systems of equations.

(2) Show that $\mathbf{x}_0$ is not already a root by verifying that $\mathbf{f}(\mathbf{x}_0) \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

(3) Using Newton's method, find a new guess $\mathbf{x}_1$ using $\mathbf{x}_0$. Calculate $\mathbf{f}(\mathbf{x}_1)$.
(You are welcome to use Matlab or a calculator to invert $\mathbf{J}(\mathbf{x})$ and perform any matrix multiplications.)

(4) Repeat this two more times by finding $\mathbf{x}_2$ and $\mathbf{x}_3$. Show that the values $\mathbf{f}(\mathbf{x}_2)$ and $\mathbf{f}(\mathbf{x}_3)$ approach $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.