BIOE 210, SPRING 2019

HOMEWORK 4A

Due Wednesday, 3/13/2019 before 9:00am.
Upload a single PDF with your answers to Gradescope.

PART I (60 POINTS)

(1) Consider the vectors $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$.
   (a) Construct an orthonormal set of basis vectors from these vectors.
   (b) Decompose the vector $\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$ onto your orthonormal basis.

You can use Matlab or a calculator to answer the following questions.

(2) Quadratic expressions can be written in the form $\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x}$.
   (a) Write the matrix $\mathbf{Q}$ for the expression

$$2x_1^2 - 4x_1x_3 + x_2x_3 + x_3^2$$

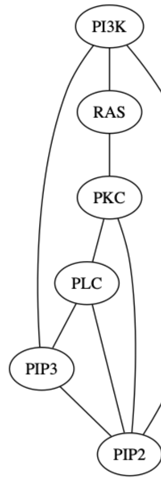
   Remember to make the matrix $\mathbf{Q}$ symmetric.
   (b) Find the eigenvalues of $\mathbf{Q}$.
   (c) Is the function $f(x_1, x_2, x_3) = 2x_1^2 - 4x_1x_3 + x_2x_3 + x_3^2$ convex?

(3) Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 5 & 3 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

   (a) Find the eigenvectors and the corresponding eigenvalues for the matrix.
   (b) Decompose the vector $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ on the eigenvectors.
   (c) Without performing a matrix multiplication, what is $\mathbf{Ax}$?

(4) The following figure depicts protein-protein interactions in a human signal transduction network. Your goal is to find the most central and least central proteins in this network.



(a) Construct an adjacency matrix to represent the connections in the network.
   - An adjacency matrix is a square matrix **A** such that each element a_{ij} equals 1 if node i is directly connected to node j.
   - Since the above graph is undirected, your adjacency matrix should be symmetric. If $a_{ij} = 1$, the $a_{ji} = 1$.
   - The diagonal elements (a_{ii}) must be left zero, since no node is "connected" to itself.

(b) Calculate the leading eigenvector for the adjacency matrix. The leading eigenvector is associated with the eigenvalue with the largest magnitude.

(c) Using the magnitude of the entries in the leading eigenvector, report the most central and least central proteins in the network. How does the centrality of these proteins compare with the number of connections involving these proteins? Is the most central protein always the protein with the largest number of direct connections?