

TAM 251: Introductory Solid Mechanics

Discussion 8

Tall-Bike Spindle Analysis

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Course

TAM 251

Submission

Discussion 8 Solution Report

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Discussion 8 — Tall-Bike Spindle Solution

Problem scope. Determine internal spindle loads, chain tensions, bearing reactions, shear-force and bending-moment diagrams, bending stresses, maximum pedal force from the specified bending criterion, and torsional shear stress.

Parameter	Value	Parameter	Value
a	70 mm	b	25 mm
c	9.5 mm	d	40 mm
L	170 mm	R	75 mm
Outer diameter, d_o	27 mm	Inner diameter, d_i	21 mm
Steel yield strength, σ_y	400 MPa	Factor of safety	2

Assumptions

- Static equilibrium is assumed.
- Bearings B and C apply forces, not moments.
- Only one pedal is loaded; the chainring is in the plane of C.
- The lower chain run is slack ($F_1 = 0$).
- Nominal linear-elastic beam bending and circular-shaft torsion formulas are used.

1. Determine internal loads at cross-section A

Using the crank-pedal free-body diagram, enforce equilibrium in y and take moments about z and x.

$$\Sigma F_y = 0: -P - V_A = 0$$

$$\Sigma M_z = 0: M_A + P(c + d) = 0$$

$$\Sigma M_x = 0: T_A - PL = 0$$

$$V_A = -P \quad M_A = -49.5P \text{ N}\cdot\text{mm} \quad T_A = 170P \text{ N}\cdot\text{mm}$$

2. Determine the chain tensions

The chain is assumed not to carry compression; the lower run is therefore slack ($F_1 = 0$).

$$\Sigma M_x = 0: (F_2 - F_1)R - T_A = 0$$

$$F_2 = T_A/R = (170/75)P = 2.267P$$

$$F_1 = 0 \quad F_2 = 2.267P$$

3. Determine the bearing reaction forces

Coordinates along the spindle are $x_A = 0$, $x_B = 25$ mm, and $x_C = 95$ mm.

$$\Sigma M_y = 0: R_{Bz} a = 0 \Rightarrow R_{Bz} = 0$$

$$\Sigma M_z C = 0: -V_A(a+b) - R_{By} a - M_A = 0$$

$$R_{By} = [-M_A - V_A(a+b)]/a = (144.5/70)P = 2.064P$$

$$\Sigma F_z = 0: R_{Bz} + R_{Cz} + F_2 = 0$$

$$\Sigma F_y = 0: V_A + R_{By} + R_{Cy} = 0$$

$$R_{By} = 2.064P \quad R_{Cy} = -1.064P \quad R_{Bz} = 0 \quad R_{Cz} = -2.267P$$

4. Shear-force and bending-moment diagrams from A to C

The reaction at B changes shear from $-P$ to $+1.064P$. The reaction at C returns it to zero. The largest absolute bending moment occurs at B.

$$0 \leq x < 25 \text{ mm: } V_y = -P; \quad M_z = -49.5P - P_x$$

$$25 \leq x \leq 95 \text{ mm: } V_y = 1.064P; \quad M_z = -74.5P + 1.064P(x - 25)$$

$$M_z(A) = -49.5P \text{ mm} \quad M_z(B) = -74.5P \text{ mm} \quad M_z(C) = 0$$

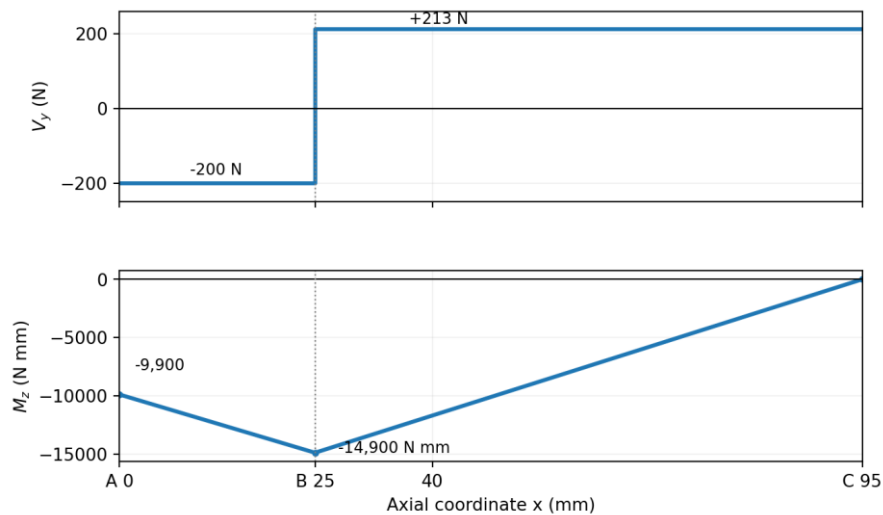


Figure 1. Shear-force and bending-moment diagrams.

5. Normal stress at H, K, and L for $P = 200 \text{ N}$ at $x = 40 \text{ mm}$

$$I_y = I_z = \pi(d_o^4 - d_i^4)/64 \approx 1.65 \times 10^4 \text{ mm}^4$$

$$M_z(40) = [-74.5 + 1.064(15)](200) \approx -11,704 \text{ N}\cdot\text{mm}$$

$$\sigma_x = -M_z y / I_z$$

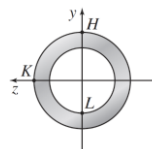


Figure 2. Hollow cross-section with H, K and L.

Point	y-coordinate	σ_x	Interpretation
H	+13.5 mm	+9.55 MPa	Tension
K	0 mm	0 MPa	Neutral axis

6. Maximum pedal force using bending-yield criterion

For the specified steel yield strength of 400 MPa and safety factor of 2, the allowable normal bending stress is:

$$\sigma_{\text{allow}} = \sigma_Y/2 = 400/2 = 200 \text{ MPa}$$

$$|M_z|_{\text{max}} = 74.5P \text{ N}\cdot\text{mm}; \quad y_{\text{max}} = d_o/2 = 13.5 \text{ mm}$$

$$|\sigma_x|_{\text{max}} = (74.5P)(13.5)/(1.65 \times 10^4) \approx 0.0608P \text{ MPa}$$

$$P_{\text{max}} \approx 200/0.0608 = 3,289 \text{ N}$$

7. Maximum torsional shear stress at P_{max}

For a hollow circular shaft, $J = I_y + I_z = 2I_z$. Peak nominal torsional shear stress is at the outer radius.

$$J \approx 2(1.65 \times 10^4) = 3.30 \times 10^4 \text{ mm}^4$$

$$T_{\text{max}} = LP_{\text{max}} = (170)(3,289) = 5.59 \times 10^5 \text{ N}\cdot\text{mm}$$

$$\tau_{\text{max}} = T_{\text{max}}(d_o/2)/J$$

$$\tau_{\text{max}} \approx 228 \text{ MPa}$$