

TAM 251

Discussion 5

Wind Turbine

Prepared by:

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Date : 09.30.2026

Signature



Checked by:

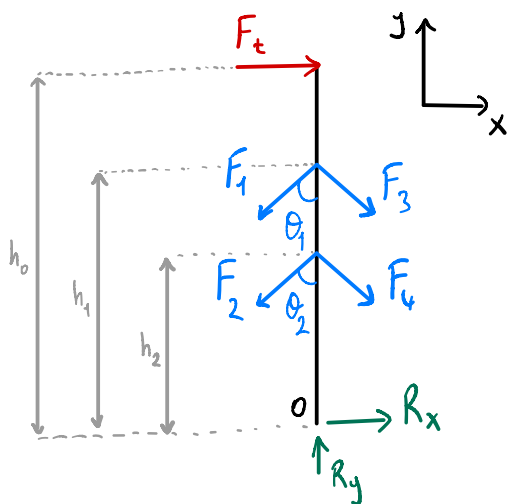
Tom Golecki

Date: 9.30.26

Signature



FB D



Assumptions

- Steady state operation
- Assume pin connections with the cables
- Pin height is negligible
- Assume small deformations
- Assume pin connection with the ground
- ⊗ • Cables are assumed as rods until the F.S. check

2) Newton's Second Law of Motion :

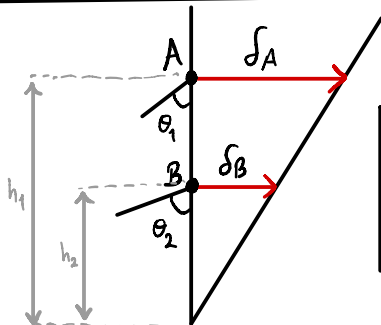
$$\sum F_x = 0 : F_t + (F_3 - F_1) \sin \theta_1 + (F_4 - F_2) \sin \theta_2 + R_x = 0 \quad (1)$$

$$\sum F_y = 0 : R_y - (F_1 + F_3) \cos \theta_1 - (F_2 + F_4) \cos \theta_2 = 0 \quad (2)$$

$$\sum M_o = 0 : -F_t h_0 + (F_1 - F_3) \sin \theta_1 h_1 + (F_2 - F_4) \sin \theta_2 h_2 = 0 \quad (3)$$

3) Deflection on the tower causes deformation on the cables

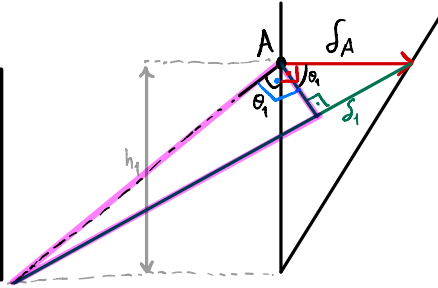
Use Triangle
Proportionality
Theorem



$$\frac{\delta_B}{\delta_A} = \frac{h_2}{h_1} \quad (4)$$

4) Relation between δ_i ($i=1,2,3,4$) and δ_A or δ_B

Small angle assumption creates isosceles triangle with 90° angles at two corners



$$\delta_1 = \delta_A \sin \theta_1$$

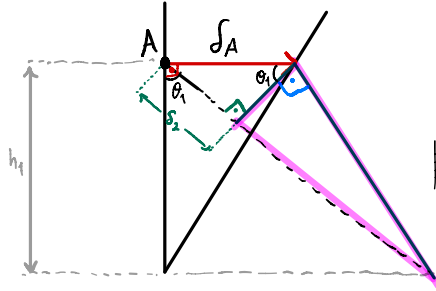
$$\delta_2 = \delta_B \sin \theta_2$$

(tension)

$$\delta_1, \delta_2 > 0$$

⊗ Absolute values yield geometric magnitudes, deliberately dropping the sign convention:

Purple highlight shows the triangle with a negligible angle



$$|\delta_3| = |\delta_A| \sin \theta_1$$

$$|\delta_4| = |\delta_B| \sin \theta_2$$

(compression)

$$\delta_3, \delta_4 < 0$$

5) Relation Between F_i & δ_i

$$F = k \delta \quad \text{same material for each} \rightarrow E = E_{1-4}$$

$$\left. \begin{array}{l} F_1 = k_1 \delta_1 \\ \vdots \\ F_4 = k_4 \delta_4 \end{array} \right\} k_{1-4} = \frac{F_{1-4}}{\delta_{1-4}}$$

* k is an extensive property
 E is an intensive property

$$E = \frac{\sigma}{\epsilon} = \frac{\frac{F}{A}}{\frac{\delta}{L}} = \frac{FL}{A\delta} = \left(\frac{F}{\delta}\right) \cdot \frac{L}{A} = k \frac{L}{A} \Rightarrow k = \frac{EA}{L} \quad (5)$$

* Same stiffness as long as E, A, L are the same

$$k_1 = k_3 \quad \& \quad k_2 = k_4 \quad (6)$$

! $k_1 \neq k_2$ due to the difference in length (L)

6) Relation Between The Forces

Using Precalculated Equations

$$\begin{cases} \delta_1 = \delta_A \sin \theta_1 & |\delta_3| = |\delta_A| \sin \theta_1 \rightarrow \delta_A = \frac{-\delta_3}{\sin \theta_1} \quad (7) \\ \delta_2 = \delta_B \sin \theta_2 & |\delta_4| = |\delta_B| \sin \theta_2 \rightarrow \delta_B = \frac{-\delta_4}{\sin \theta_2} \quad (8) \end{cases}$$

$$\delta_1 = \frac{-\delta_3}{\cancel{\sin \theta_1}} \cancel{\sin \theta_1} \rightarrow \delta_1 = -\delta_3 \quad \begin{matrix} 2 \text{ \& } 4 \text{ are} \\ \text{calculated} \\ \text{in the} \\ \text{same} \\ \text{way} \end{matrix}$$

$$L_1 = L_3$$

$$k_1 = \frac{EA}{L_1} = k_3$$

Equal in magnitude
opposite in direction

$$F_1 = k_1 \delta_1 = k_3 (-\delta_3) = -F_3$$

$$\begin{aligned} \delta_2 &= -\delta_4 \\ L_2 &= L_4 \\ k_2 &= k_4 \\ F_2 &= -F_4 \end{aligned}$$

From previous Calculations:

$$L_1 = \sqrt{w^2 + h_1^2} \quad L_2 = \sqrt{w^2 + h_2^2}$$

$$\frac{\delta_B}{\delta_A} = \frac{h_2}{h_1} \Rightarrow \frac{\delta_1 = \delta_A \sin \theta_1}{\delta_2 = \delta_B \sin \theta_2} \rightarrow \frac{\delta_B}{\delta_A} = \frac{\delta_2 \cdot \sin \theta_1}{\delta_1 \sin \theta_2} = \frac{h_2}{h_1}$$

$$\frac{\delta_1}{\delta_2} = \frac{h_1 \sin \theta_1}{h_2 \sin \theta_2} \quad (9)$$

Ratio of
stiffness
values

$$\frac{k_1 = \frac{EA}{L_1}}{k_2 = \frac{EA}{L_2}} = \frac{L_2}{L_1} = \sqrt{\frac{w^2 + h_2^2}{w^2 + h_1^2}}$$

Ratio of
forces

$$\frac{F_1}{F_2} = \frac{k_1 \delta_1}{k_2 \delta_2} = \frac{L_2}{L_1} \cdot \frac{h_1 \sin \theta_1}{h_2 \sin \theta_2} \quad (10)$$

7) Check F.S.

⊗ Drop the rod assumption for the cables →

$$F_1 \neq -F_3$$

$$F_2 \neq -F_4$$

$$F_3 = F_4 = 0$$

↳ Real cables buckle

Unknowns
stress values
deflection values
force values

Solution
direction

$$\sigma = E \epsilon = E \frac{\delta}{L}$$

$$F = k \delta$$

$$\sum M_o = 0$$

$$\frac{F_1}{F_2} = \frac{L_2}{L_1} \cdot \frac{h_1 \sin \theta_1}{h_2 \sin \theta_2}$$

$$-F_t h_o + (F_1 - \cancel{F_3}) \sin \theta_1 h_1 + (F_2 - \cancel{F_4}) \sin \theta_2 h_2 = 0$$

$$-F_t h_o + \left[F_2 \frac{L_2}{L_1} \cdot \frac{h_1 \sin \theta_1}{h_2 \sin \theta_2} \right] \sin \theta_1 h_1 + F_2 \sin \theta_2 h_2 = 0$$

$$-F_t h_o + F_2 \left(\left[\frac{L_2}{L_1} \cdot \frac{h_1 \sin \theta_1}{h_2 \sin \theta_2} \right] \sin \theta_1 h_1 + \sin \theta_2 h_2 \right) = 0$$

Using SI Units

$$-(1112 \text{ N}) \cdot (13.72 \text{ m}) + F_2 \left(\left[\frac{11.34 \text{ m}}{14.52 \text{ m}} \cdot \frac{11.28 \text{ m} \sin(0.681)}{6.706 \text{ m} \sin(0.938)} \right] \sin(0.681) (11.28 \text{ m}) + \sin(0.938) (6.706 \text{ m}) \right) = 0$$

Use eq (10):

$$F_2 = 1201 \text{ N} \rightarrow F_1 = 1232 \text{ N}$$

$$\delta_1 = \frac{F_1}{k_1} = \frac{F_1 L_1}{E A} = \frac{(1232 \text{ N}) (14.52 \text{ m})}{(200 \cdot 10^9 \text{ Pa}) \cdot \frac{\pi (6.35 \cdot 10^{-3} \text{ m})^2}{4}} = 2.83 \times 10^{-3} \text{ m}$$

↳ small deflection
assumption is
correct

Use eq (9)

$$\delta_2 = \delta_1 \frac{h_2 \sin \theta_2}{h_1 \sin \theta_1} = 2.152 \times 10^{-3} \text{ m} \\ \approx \underline{2.2 \times 10^{-3} \text{ m}}$$

$F = L \delta$ ✓

$$\sigma_1 = E \frac{\delta_1}{L_1} = (200 \cdot 10^9 \text{ Pa}) \frac{2.83 \times 10^{-3} \text{ m}}{14.52 \text{ m}} = 38.92 \text{ MPa} \\ \approx \underline{39 \text{ MPa}}$$

$$\sigma_2 = E \frac{\delta_2}{L_2} = (200 \cdot 10^9 \text{ Pa}) \frac{2.152 \times 10^{-3} \text{ m}}{11.34 \text{ m}} = 37.94 \text{ MPa} \\ \approx \underline{38 \text{ MPa}}$$

2 significant

digits in the
problem parameters

don't provide
more in the answer

$$FS_1 = \frac{\sigma_y}{\sigma_1} = \frac{248 \text{ MPa}}{38.92 \text{ MPa}} \approx 6 \gg 2$$

It is too
safe
(waste of resources)

$$\otimes \sigma_1 > \sigma_2 \text{ so } FS_2 = \frac{\sigma_y}{\sigma_2} > FS_1 = \frac{\sigma_y}{\sigma_1}$$

No need to calculate FS_2 .

8) Were the assumptions accurate?

Cables have no compressive load capability. This solution did not assume cables as rods. If we were assuming cables as rods, we would have less conservative solution that would yield a higher F.S.

9) Temperature consideration?

In hot weather cables will elongate and the tower can be tilted.
In cold weather cables will shrink and might snap due to high stress.
Calculations for changing environmental conditions should also be conducted

APPENDIX

```
import math
```

```
import numpy as np
```

```
# =====
```

```
# 1. INPUT VALUES (US Customary Units)
```

```
# =====
```

```
# Dimensions
```

```
w_ft = 30.0 # ft
```

```
h0_ft = 45.0 # ft
```

```
h1_ft = 37.0 # ft
```

```
h2_ft = 22.0 # ft
```

```
# Steel Cable Properties
```

```
E_s_ksi = 29e3 # ksi (10^3 psi)
```

```
sigma_y_ksi = 36.0 # ksi
```

```
d_in = 0.25 # in
```

```
alpha_per_F = 7e-6 # 1/°F
```

```
# Other Design Variables
```

```
FS = 2.0 # dimensionless
```

```
Ft_lb = 250.0 # lb (force)
```

```
# =====
```

```
# 2. CONVERSION FACTORS TO SI UNITS
```

```
# =====
```

FT_TO_M = 0.3048 # 1 ft = 0.3048 m

IN_TO_M = 0.0254 # 1 in = 0.0254 m

KSI_TO_PA = 6.894757293e6 # 1 ksi = 6,894,757.293 Pa (N/m²)

LB_TO_N = 4.4482216152605 # 1 lbf = 4.44822 N

DELTA_F_TO_DELTA_K = 5.0 / 9.0 # 1 /°F = (9/5) /°C or /K

=====

3. UNIT CONVERSIONS

=====

Lengths & Diameters (m)

w_m = w_ft * FT_TO_M

h0_m = h0_ft * FT_TO_M

h1_m = h1_ft * FT_TO_M

h2_m = h2_ft * FT_TO_M

d_m = d_in * IN_TO_M

Cross-sectional area (m²)

area_m2 = math.pi * (d_m / 2.0) ** 2

Stresses & Moduli (Pa, MPa, GPa)

E_s_Pa = E_s_ksi * KSI_TO_PA

E_s_GPa = E_s_Pa / 1e9

sigma_y_Pa = sigma_y_ksi * KSI_TO_PA

sigma_y_MPa = sigma_y_Pa / 1e6

```
# Thermal expansion coefficient (1/K or 1/°C)
```

```
# Note: For changes/gradients, 1 /°F = (9/5) /K
```

```
alpha_per_K = alpha_per_F * (9.0 / 5.0)
```

```
# Thrust force (N)
```

```
Ft_N = Ft_lb * LB_TO_N
```

```
# =====
```

```
# 4. PRINT CONVERSION TABLE
```

```
# =====
```

```
results = [
```

```
    ("Width (w)", f"{w_ft} ft", f"{w_m:.4f} m"),
```

```
    ("Height h0", f"{h0_ft} ft", f"{h0_m:.4f} m"),
```

```
    ("Height h1", f"{h1_ft} ft", f"{h1_m:.4f} m"),
```

```
    ("Height h2", f"{h2_ft} ft", f"{h2_m:.4f} m"),
```

```
    ("Cable diameter (d)", f"{d_in} in", f"{d_m * 1e3:.3f} mm ({d_m:.5f} m)",
```

```
    ("Cable area (A)", f"~0.0491 in2", f"{area_m2 * 1e6:.3f} mm2",
```

```
    (
```

```
        "Young's modulus (Es)",
```

```
        f"{E_s_ksi:g} ksi",
```

```
        f"{E_s_GPa:.2f} GPa ({E_s_Pa:.3e} Pa)",
```

```
    ),
```

```
    (
```

```
        "Yield strength (σy)",
```

```
        f"{sigma_y_ksi} ksi",
```

```
        f"{sigma_y_MPa:.2f} MPa ({sigma_y_Pa:.3e} Pa)",
```

```

),
("Thermal expansion ( $\alpha$ )", f"{alpha_per_F:.1e} /°F", f"{alpha_per_K:.2e} /K"),
("Thrust force (Ft)", f"{Ft_lb} lb", f"{Ft_N:.2f} N"),
("Factor of safety (FS)", f"{FS}", f"{FS} (dimensionless)"),
]

```

```

print(f"{'Parameter':<25} | {'Original (US)':<18} | {'Converted (SI)':<30}")
print("-" * 80)
for param, orig, si in results:
    print(f"{param:<25} | {orig:<18} | {si:<30}")

```

```

L1 = np.sqrt(w_m**2 + h1_m**2)
L2 = np.sqrt(w_m**2 + h2_m**2)
theta1_rad = np.arctan(w_m/h1_m)
theta2_rad = np.arctan(w_m/h2_m)

```

```

Placeholder_A = (L2*h1_m*np.sin(theta1_rad))/(L1*h2_m*np.sin(theta2_rad))
Placeholder_B = np.sin(theta1_rad)*h1_m
Placeholder_C = np.sin(theta2_rad)*h2_m
F2_N = Ft_N*h0_m/(Placeholder_A*Placeholder_B + Placeholder_C)
F1_N = F2_N*Placeholder_A

```

```

print(F1_N)
print(F2_N)

```

```
del_1 = F1_N*L1/(E_s_Pa*area_m2)
```

```
print(del_1)
```

```
del_2 = del_1*np.sin(theta2_rad)*h2_m/(np.sin(theta1_rad)*h1_m)
```

```
print(del_2)
```

```
sigma1_Pa = E_s_Pa*del_1/L1
```

```
sigma2_Pa = E_s_Pa*del_2/L2
```

```
FS= sigma_y_Pa/sigma1_Pa
```

```
print(sigma1_Pa*1e-6)
```

```
print(sigma2_Pa*1e-6)
```

```
print(FS)
```

```
import numpy as np
```

```
#
```

```
=====
```

```
=====
```

```
# 1. CABLE GEOMETRY
```

```
#
```

```
=====
```

```
=====
```

```
# Lengths of cables 1 and 2 using Pythagorean theorem
```

```
L1 = np.sqrt(w_m**2 + h1_m**2)
```

```
L2 = np.sqrt(w_m**2 + h2_m**2)
```

```
# Cable angles relative to the vertical post
```

```
theta1_rad = np.arctan(w_m / h1_m)
```

```
theta2_rad = np.arctan(w_m / h2_m)
```

```
#
```

```
=====
```

```
# 2. INDETERMINATE SYSTEM SOLUTION (Compatibility + Equilibrium)
```

```
#
```

```
=====
```

```
# Cable force ratio from geometric compatibility:
```

```
# Rotation theta causes cable elongations  $\delta = h \cdot \sin(\theta_{\text{cable}}) \cdot \theta_{\text{post}}$ 
```

```
# and  $\delta = F \cdot L / (A \cdot E)$ , leading to:  $F_1 / F_2 = (L_2 \cdot h_1 \cdot \sin(\theta_1)) / (L_1 \cdot h_2 \cdot \sin(\theta_2))$ 
```

```
force_ratio_F1_to_F2 = (L2 * h1_m * np.sin(theta1_rad)) / (
```

```
    L1 * h2_m * np.sin(theta2_rad)
```

```
)
```

```
# Moment arm contributions about the post base:
```

```
moment_arm_cable1 = np.sin(theta1_rad) * h1_m
```

```
moment_arm_cable2 = np.sin(theta2_rad) * h2_m
```

```
# Moment equilibrium about base:  $\sum(M) = F_t \cdot h_0 - (F_1 \cdot \text{moment\_arm}_1 + F_2 \cdot \text{moment\_arm}_2) = 0$ 
```

```
# Substituting F1 = F2 * force_ratio allows solving directly for F2:
```

```
F2_N = (Ft_N * h0_m) / (  
    force_ratio_F1_to_F2 * moment_arm_cable1 + moment_arm_cable2  
)  
F1_N = F2_N * force_ratio_F1_to_F2
```

```
#  
=====
```

```
# 3. DEFLECTIONS / ELONGATIONS (delta = F * L / (A * E))
```

```
#  
=====
```

```
del_1 = (F1_N * L1) / (E_s_Pa * area_m2)
```

```
# Elongation of cable 2 from geometric compatibility
```

```
del_2 = (  
    del_1  
    * (np.sin(theta2_rad) * h2_m)  
    / (np.sin(theta1_rad) * h1_m)  
)
```

```
#  
=====
```

```
# 4. NORMAL STRESSES & FACTOR OF SAFETY
```

```

#
=====
=====

# Stress ( $\sigma = E * \epsilon = E * \delta / L = F / A$ )

sigma1_Pa = E_s_Pa * del_1 / L1
sigma2_Pa = E_s_Pa * del_2 / L2


# Critical factor of safety (governed by the more heavily stressed cable)

critical_sigma_Pa = max(sigma1_Pa, sigma2_Pa)

FS = sigma_y_Pa / critical_sigma_Pa


#
=====
=====

# 5. FORMATTED OUTPUT

#
=====
=====

print("=" * 60)

print("      STRUCTURAL ANALYSIS RESULTS (SI UNITS)      ")

print("=" * 60)


print("\n--- Cable Geometry ---")

print(
    f"Cable 1 Length (L1) : {L1:8.3f} m | Angle: {np.degrees(theta1_rad):6.2f}°"
)

print(
    f"Cable 2 Length (L2) : {L2:8.3f} m | Angle: {np.degrees(theta2_rad):6.2f}°"
)

```

)

print("\n--- Cable Tensile Forces ---")

print(

f"Cable 1 Force (F1) : {F1_N:8.2f} N ({F1_N / 1e3:6.3f} kN)"

)

print(

f"Cable 2 Force (F2) : {F2_N:8.2f} N ({F2_N / 1e3:6.3f} kN)"

)

print("\n--- Cable Elongations ---")

print(

f"Cable 1 Elongation (δ_1): {del_1 * 1e3:8.3f} mm ({del_1:10.4e} m)"

)

print(

f"Cable 2 Elongation (δ_2): {del_2 * 1e3:8.3f} mm ({del_2:10.4e} m)"

)

print("\n--- Normal Stresses & Safety Margin ---")

print(f"Cable 1 Stress (σ_1) : {sigma1_Pa * 1e-6:8.2f} MPa")

print(f"Cable 2 Stress (σ_2) : {sigma2_Pa * 1e-6:8.2f} MPa")

print(f"Yield Strength (σ_y) : {sigma_y_Pa * 1e-6:8.2f} MPa")

print(f"Computed Factor of Safety (FS): {FS:6.2f}")

status = (

"PASS (FS >= 2.0)"

```
if FS >= 2.0
else "FAIL (FS < 2.0 - design criteria not met)"
)
print(f"Design Verification : {status}")
print("=" * 60)
```

Full transcript of the Generative AI usage:
<https://share.gemini.google/BFPsenX52iFo>