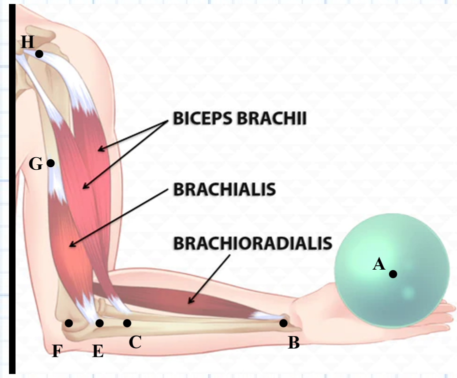
University of Illinois Urbana-Champaign
Theoretical and Applied Mechanics

Class: TAM 251
Assignment: D2

Originated by: TFG Signed: [signature] Date: 7/15/26
Checked by: EY Signed: [signature] Date: 8/31/26

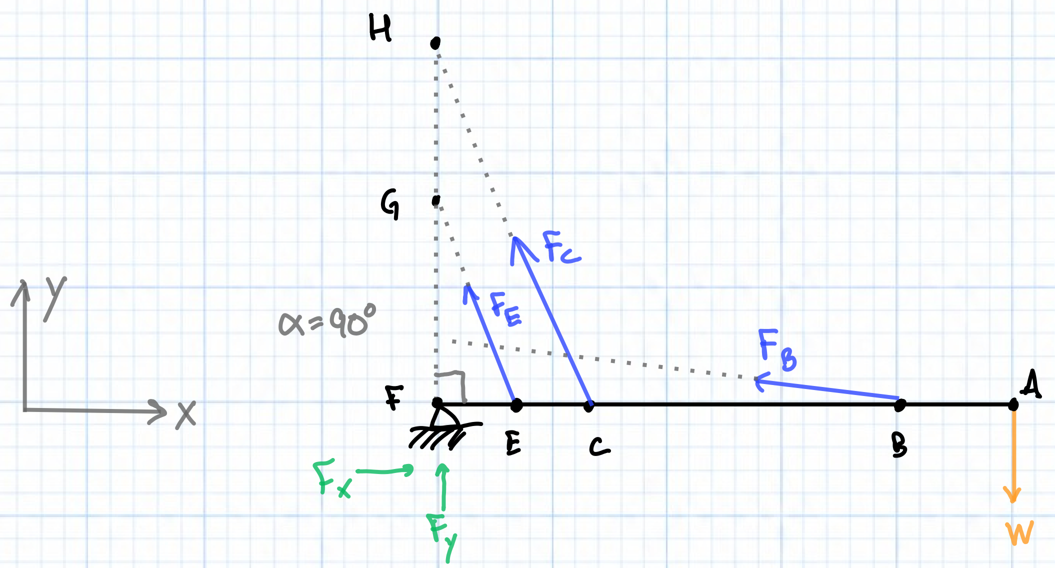
This calculation documents the forces in the biceps as a function of the forearm angle.



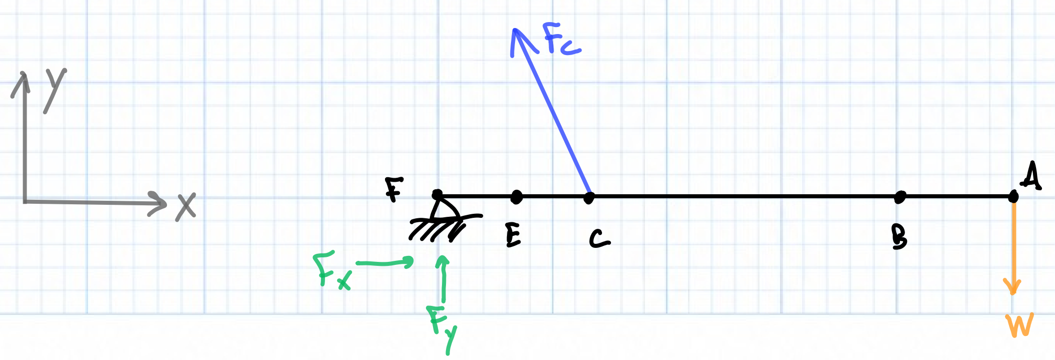
1) Idealization of components and loads on the system.

Treat forearm as a beam with hinge connection at the elbow. The upper arm (bone) is not included in the model. Muscles are treated as forces from points E, C toward points G, H. Weight is treated as vertical downward force at A. Assumptions; Only static loading (no dynamics). Assume the position of the elbow does not change (pin support)

2) Free Body Diagram



3) As idealized here, this system has no boundary condition to prevent rotation about F, making the system unstable (neither determinate or indeterminate). This system has three muscles which can contribute to resisting rotation about point F, so without additional assumptions we cannot determine the magnitudes of these forces. Assume only the biceps are resisting rotation. (Assume a negligible contribution from the other muscles.)



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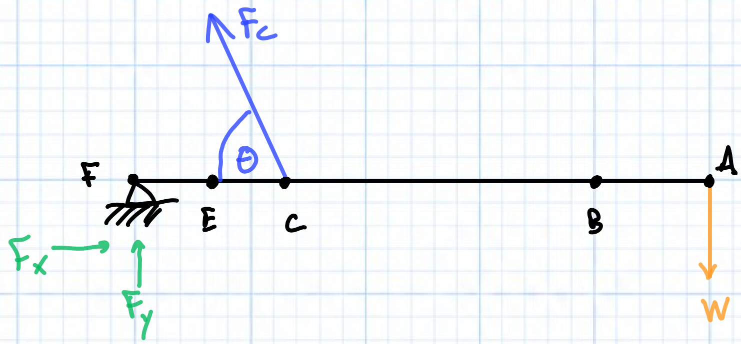
4) Find expression for force in biceps as a function of elbow rotation angle α. Assume force is always directed toward point H.



$$\Sigma M_F: \quad l_{FC} F_c \sin\theta - l_{FA} W = 0$$

$$F_c = \frac{l_{FA}}{l_{FC}} \frac{W}{\sin\theta}$$

5) Stress in biceps assuming θ is 75°, length FC is 70mm and length CB is 230mm and cross sectional area is 194 mm^2. (Length BA is not known, for the purpose of this estimate, assume this length is negligibly small.)

$$F_c = \left(\frac{70+230}{70}\right)\left(\frac{20}{\sin(75)}\right)$$

$$F_c = 88.7\ N$$

$$\sigma_c = \frac{F_c}{A_c} = \frac{88.7N}{194\ mm^2} = 0.457\ \frac{N}{mm^2}$$

$$= 457\ kPa$$

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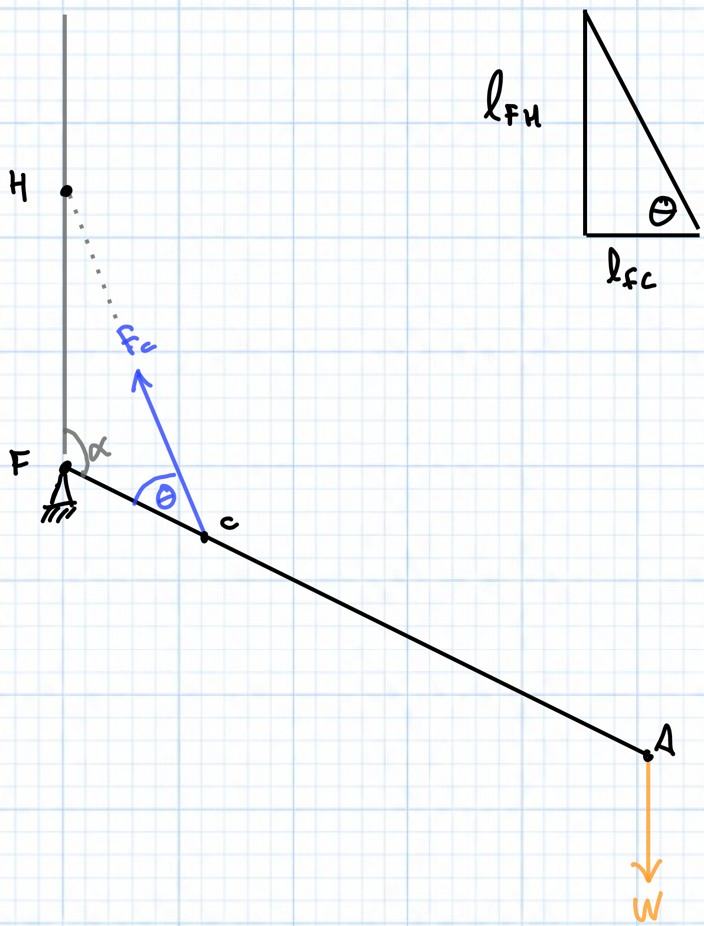
6) Repeat the stress calculation allowing the elbow to rotate.



$$\tan 75 = \frac{l_{FH}}{l_{FC}}$$

$$l_{FH} = l_{FC} \tan(75^\circ)$$

LAW OF SINES:

$$\frac{l_{FH}}{\sin\theta} = \frac{l_{CH}}{\sin\alpha}$$

$$\frac{\sin\alpha}{\sin\theta} = \frac{l_{CH}}{l_{FH}}$$

$$\Sigma M_F: \quad l_{FC} F_c (\sin\theta) - l_{FA} W (\sin\alpha) = 0$$

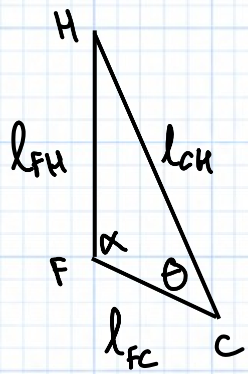
$$F_c(\alpha) = \frac{l_{FA}}{l_{FC}} W \frac{\sin\alpha}{\sin\theta}$$

SUBSTITUTE FROM LAW OF SINES

$$F_c(\alpha) = \frac{l_{FA}}{l_{FC}} W \frac{l_{CH}}{l_{FH}}$$

NEED TO FIND l_{CH} AS FUNCTION OF α

LAW OF COSINES



$$l_{CH}^2 = l_{FC}^2 + l_{FH}^2 - 2\, l_{FC}\, l_{FH} \cos\alpha$$

SUBSTITUTE $l_{FH} = l_{FC} \tan(75°)$

$$l_{CH}^2 = l_{FC}^2 + (l_{FC} \tan 75)^2 - 2\, l_{FC}\, l_{FC} \tan 75 \cos\alpha$$

$$l_{CH}^2 = l_{FC}^2 \left[1 + (\tan 75°)^2 - 2(\tan 75°)\cos\alpha\right]$$

$$l_{CH} = l_{FC}\left[1 + (\tan 75°)^2 - 2(\tan 75°)\cos\alpha\right]^{1/2}$$

$$F_C(\alpha) = \frac{l_{FA}}{l_{FC}}\, W\, \frac{l_{CH}}{l_{FH}}$$

$$F_C(\alpha) = \frac{l_{FA}}{l_{FC}}\, W\, \frac{\cancel{l_{FC}}\left[1 + (\tan 75°)^2 - 2(\tan 75°)\cos\alpha\right]^{1/2}}{\cancel{l_{FC}} \tan 75°}$$

$$\sigma_C(\alpha) = \frac{F_C(\alpha)}{A_C} = \frac{l_{FA}}{l_{FC}}\, \frac{W}{A_C}\, \frac{\left[1 + (\tan 75°)^2 - 2(\tan 75°)\cos\alpha\right]^{1/2}}{\tan 75°}$$

$l_{FA} = 300\text{ mm}$

$l_{FC} = 70\text{ mm}$

$W = 20\text{ N}$

$A_C = 194\text{ mm}^2$

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syms alpha
theta = deg2rad(75);
sigma = (300/70)*(20/(194*10^-6))*((1+
(tan(theta))^2-2*tan(theta)*cos(deg2rad(alpha)))^0.5/(tan(theta)));
fplot(sigma/1000,[45,180],'LineWidth', 2.5);
ylim([0,600])
title({"Stress in biceps as a function of elbow angle", "(90° is horizontal, 180°
is down)"})
xlabel("Angle (deg)")
ylabel("Stress (kPa)")
grid on
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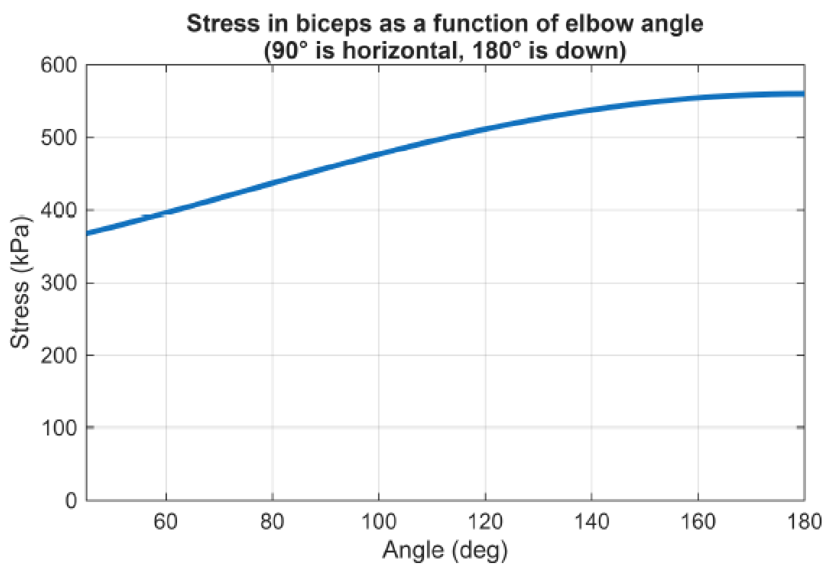

7) If the cross sectional area were different, the stress would also be different. Because the bicep has the same force along its entire length in this model, any reduction in cross sectional area would result in higher stress at that location. If the capacity of the muscle is constant, then a muscle tear is most likely to occur where the cross sectional area is lowest since this location would have the highest stress.