TAM 251

Discussion 11

Compressed Air Tanks

Prepared By:	**Checked By:**
Yigit Hergul	Nusrat Jahan Salim
Date:	**Date:**
10.27.2026	11.12.2026
Signature:	**Signature:**
Yigit	Nusrat

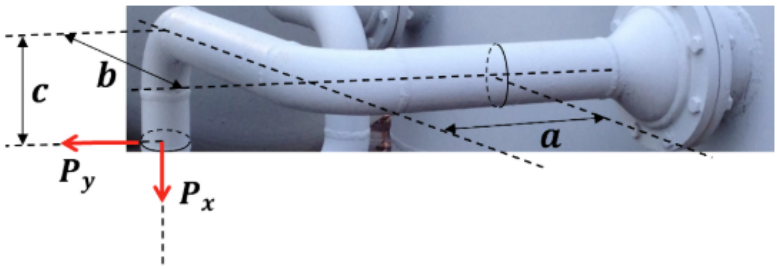


Figure 1: Connection dimensions of the pipe system.

Given Parameters and Cross-Section Properties

- **Dimensions:** $a = 3$ ft $= 36$ in, $\quad b = 2$ ft $= 24$ in, $\quad c = 1$ ft $= 12$ in
- **Loads at section A:**

$$P_x = -2.5 \text{ kips} \quad (\text{downward}, \ -x), \qquad P_y = +1.0 \text{ kip} \quad (\text{tension}, \ +y)$$

- **Pipe dimensions:** $d_o = 8$ in $(r_o = 4$ in$)$, $\quad d_i = 7.5$ in, $\quad$ wall thickness $t_w = 0.25$ in $(t = 2t_w = 0.50$ in$)$
- **Geometric properties (Table 1):**

$$A = 6.09 \text{ in}^2, \quad I = 45.7 \text{ in}^4, \quad J = 91.4 \text{ in}^4, \quad Q = 7.51 \text{ in}^3$$

- **Location K:** $(x = 0, \ y = 0, \ z = +4.0$ in$)$ on the outer pipe surface.

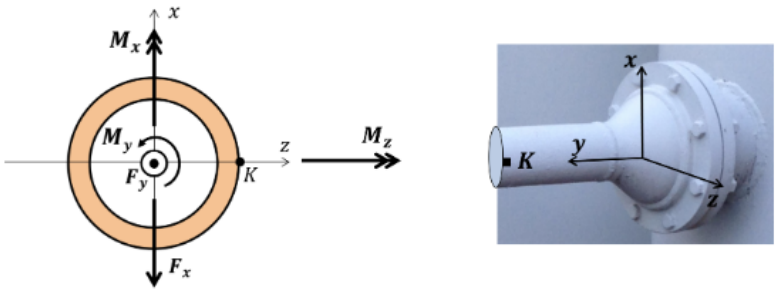


Figure 2: Cross section showing location K.

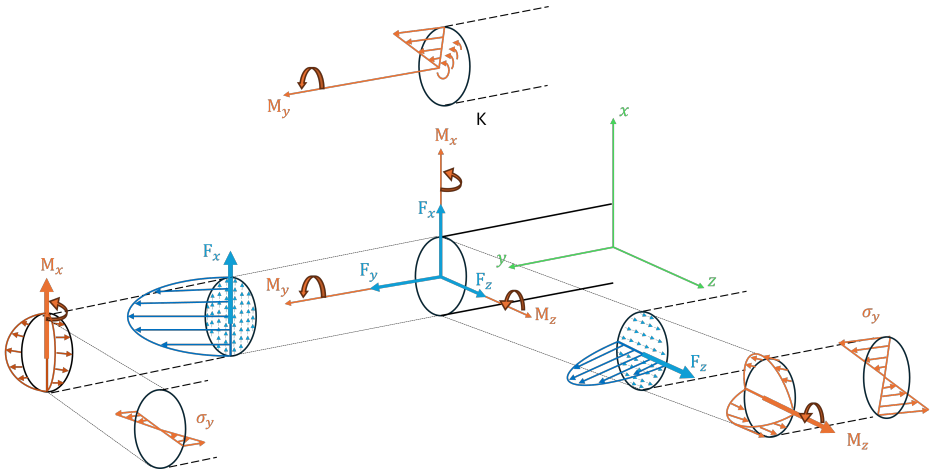


Figure 3: The normal and shear stresses in the cross sections.

Step 1: Expressions Relating Section Loads to Pipe Stresses

As shown in Figure 3, for a cut section with outward normal pointing in the $+y$ direction:

1. **Axial Force (F_y):** $\sigma_y = \dfrac{F_y}{A}$
2. **Transverse Shear in x (F_x):** $\tau_{yx} = \dfrac{F_x Q_z}{I_z t}$
3. **Transverse Shear in z (F_z):** $\tau_{yz} = \dfrac{F_z Q_x}{I_x t}$
4. **Torsional Torque ($M_y = T$):** $\tau = \dfrac{Tr}{J}$
5. **Bending Moment about x (M_x):** $\sigma_y = -\dfrac{M_x z}{I_x}$
6. **Bending Moment about z (M_z):** $\sigma_y = +\dfrac{M_z x}{I_z}$

Step 2: Internal Forces and Moments at Cross Section B

Consider the free-body diagram in Figure 4 to the load application point A.

Note about Figure 4, force and moment components are tentatively placed as positive for the negative face and corresponding negative vectors placed at the positive face.

The position vector from cut section B to point A is:

$$\mathbf{r}_{B\to A} = L_x\,\hat{i} + L_y\,\hat{j} + L_z,\hat{k}, \qquad \text{where } L_x = -c = -1 \text{ ft}, \quad L_y = a = +3 \text{ ft}, \quad L_z = -b = -2 \text{ ft}$$

The external applied force vector at point A is:

$$\mathbf{P} = P_x\,\hat{i} + P_y\,\hat{j} + P_z\,\hat{k}, \qquad \text{where } P_x = -2.5 \text{ kips}, \quad P_y = +1.0 \text{ kip}, \quad P_z = 0$$

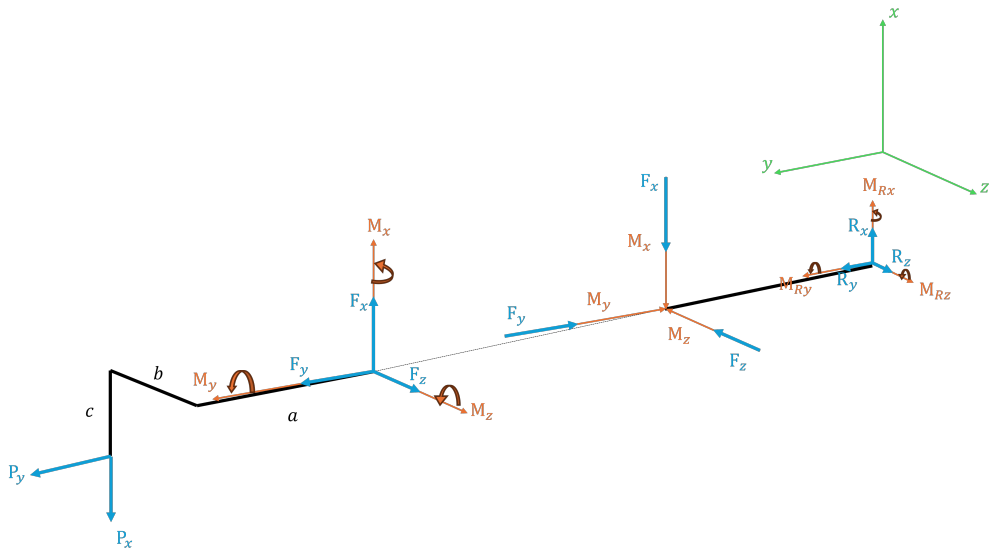


Figure 4: Free-body diagram of the pipe system with a section cut at cross-section B.

Newton's Second Law (Static Equilibrium)

For the cut section to be in static equilibrium, the net external forces and net moments about point B must vanish:

$$\sum \mathbf{F} = \mathbf{0} \qquad \text{and} \qquad \sum \mathbf{M}_B = \mathbf{0}$$

1. Force Balance Equations

Applying $\sum F_x = 0$, $\sum F_y = 0$, and $\sum F_z = 0$ for the negative face in Figure 4:

$$\sum F_x = 0: \qquad F_x - P_x = 0 \qquad \Longrightarrow \; F_x = P_x = \mathbf{2.5} \text{ kips}$$

$$\sum F_y = 0: \qquad F_y + P_y = 0 \qquad \Longrightarrow \; F_y = -P_y = \mathbf{-1.0} \text{ kip}$$

$$\sum F_z = 0: \qquad F_z + P_z = 0 \qquad \Longrightarrow \; F_z = -P_z = \mathbf{0} \text{ kips}$$

2. Moment Balance Equations about Section B

Summing moments about point B for the negative face in Figure 4 using the right-hand rule :

$$\mathbf{M}_B = \mathbf{r}_{B\to A} \times \mathbf{P}$$

$$\hat{i} \times \hat{j} = \hat{k}, \qquad \hat{j} \times \hat{k} = \hat{i}, \qquad \hat{k} \times \hat{i} = \hat{j}$$

$$\sum M_{B,x} = 0: \quad M_x + [(L_y)(P_z) - (L_z)(P_y)] = 0$$
$$M_x = -(a)(0) + (-b)(P_y) = -b \cdot P_y$$
$$M_x = -(2 \text{ ft})(1.0 \text{ kip}) = -2.0 \text{ kip} \cdot \text{ft} = \mathbf{-24.0} \text{ kip} \cdot \text{in}$$

$$\sum M_{B,y} = 0: \quad M_y + [(L_z)(P_x) - (L_x)(P_z)] = 0$$
$$M_y = T = -(-b)(P_x) + (-c)(0) = b \cdot P_x$$
$$M_y = T = -(-2 \text{ ft})(-2.5 \text{ kips}) = -5.0 \text{ kip} \cdot \text{ft} = \mathbf{-60.0} \text{ kip} \cdot \text{in}$$

$$\sum M_{B,z} = 0: \quad M_z + [(L_x)(P_y) - (L_y)(P_x)] = 0$$
$$M_z = -(-c)(P_y) + (a)(P_x)$$
$$M_z = -(-1 \text{ ft})(1.0 \text{ kip}) + (3 \text{ ft})(-2.5 \text{ kips})$$
$$M_z = 1.0 - 7.5 = -6.5 \text{ kip} \cdot \text{ft} = \mathbf{-78.0} \text{ kip} \cdot \text{in}$$

Note that these signs are for the force and moment vectors given in Figure 4, and the negative signs indicate that the assigned directions are incorrect. Let's define the correct directions to be used in formulas as:

Positive Face (outward normal in $+y$):

$$\sum F_x = 0: \qquad F_x^{(+)} + F_x^{(-)} = 0 \qquad \Longrightarrow \; F_x^{(+)} = -F_x^{(-)} = -(2.5 \text{ kips}) = \mathbf{-2.5} \text{ kips}$$

$$\sum F_y = 0: \qquad F_y^{(+)} + F_y^{(-)} = 0 \qquad \Longrightarrow \; F_y^{(+)} = -F_y^{(-)} = -(-1.0 \text{ kip}) = \mathbf{1.0} \text{ kip}$$

$$\sum F_z = 0: \qquad F_z^{(+)} + F_z^{(-)} = 0 \qquad \Longrightarrow \; F_z^{(+)} = -F_z^{(-)} = -(0 \text{ kips}) = \mathbf{0} \text{ kips}$$

$$\sum M_{B,x} = 0: \quad M_x^{(+)} + M_x^{(-)} = 0 \quad \Longrightarrow \; M_x^{(+)} = -M_x^{(-)} = -(-24.0 \text{ kip} \cdot \text{in}) = +\mathbf{24.0} \text{ kip} \cdot \text{in}$$

$$\sum M_{B,y} = 0: \quad M_y^{(+)} + M_y^{(-)} = 0 \quad \Longrightarrow \; M_y^{(+)} = -M_y^{(-)} = -(-60.0 \text{ kip} \cdot \text{in}) = +\mathbf{60.0} \text{ kip} \cdot \text{in}$$

$$\sum M_{B,z} = 0: \quad M_z^{(+)} + M_z^{(-)} = 0 \quad \Longrightarrow \; M_z^{(+)} = -M_z^{(-)} = -(-78.0 \text{ kip} \cdot \text{in}) = +\mathbf{78.0} \text{ kip} \cdot \text{in}$$

Step 3: Normal Stresses at Location K ($x = 0$, $z = +4$ in)

- **From Axial Force F_y:**

$$\sigma_y = \frac{F_y^{(+)}}{A} = \frac{+1.0 \text{ kip}}{6.09 \text{ in}^2} = +0.164 \text{ ksi} \quad \text{(Tension)}$$

- **From Bending Moment M_x:**

$$\sigma_y = -\frac{M_x^{(+)} z}{I} = -\frac{(24.0 \text{ kip} \cdot \text{in})(4.0 \text{ in})}{45.7 \text{ in}^4} = -2.101 \text{ ksi} \quad \text{(Compression)}$$

- **From Bending Moment M_z:** Since point K is on the neutral axis ($x = 0$):

$$\sigma_y = \frac{M_z^{(+)}(0)}{I} = 0 \text{ ksi}$$

Step 4: Shear Stresses at Location K ($x = 0$, $z = +4$ in)

- **From Transverse Shear F_x:** Point K is on the neutral axis of bending for F_x, where transverse shear is maximum. Since F_x points downward $(-x)$, the transverse shear is downward:

$$\tau_{yx}^{(V)} = \frac{|F_x^{(+)}|Q}{It} = \frac{(2.5 \text{ kips})(7.51 \text{ in}^3)}{(45.7 \text{ in}^4)(0.50 \text{ in})} = 0.822 \text{ ksi} \quad (\text{downward}, \ -x)$$

- **From Torsional Torque $T = M_y$:** By the right-hand rule about $+y$, torque produces upward tangential shear at $z = +r_o$:

$$\tau^{(T)} = \frac{Tr_o}{J} = \frac{(60.0 \text{ kip} \cdot \text{in})(4.0 \text{ in})}{91.4 \text{ in}^4} = 2.626 \text{ ksi} \quad (\text{upward}, \ +x)$$

- **From Transverse Shear F_z:** $\tau = 0$ because $F_z = 0$.

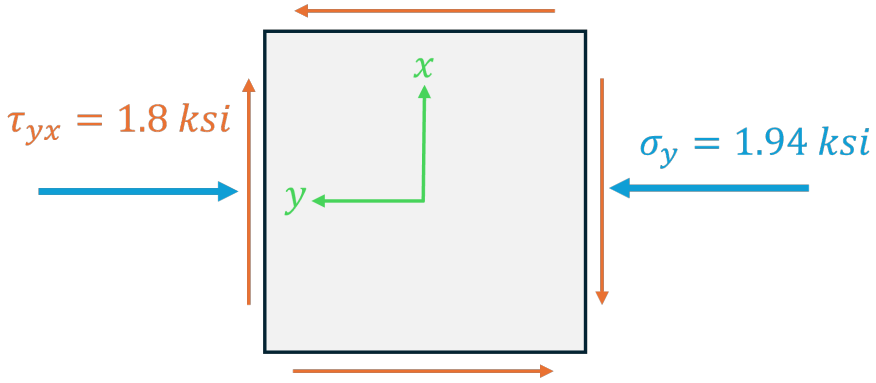


Figure 5: Combined Stresses on the Stress Element.

Step 5: Total Combined Stresses and Stress Element

Superimposing normal and shear stresses at point K:

$$\sigma_y = +0.164 - 2.101 + 0 = -1.937 \text{ ksi} \quad (\text{Compression})$$

$$\sigma_x = 0$$

$$\tau_{yx} = \tau^{(T)} - \tau_{yx}^{(V)} = 2.626 - 0.822 = +1.804 \text{ ksi} \quad (\text{upward on } +y \text{ face})$$

Step 6: Principal Stresses and Orientation

With $\sigma_x = 0$, $\sigma_y = -1.937$ ksi, and $\tau_{xy} = 1.804$ ksi:

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-1.937)}{2} = -0.968 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(0.968)^2 + (1.804)^2} = 2.047 \text{ ksi}$$

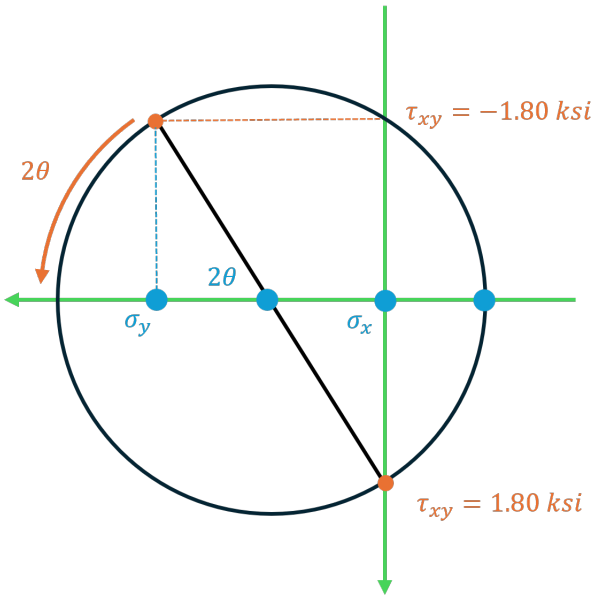


Figure 6: Mohr's Circle.

Principal Stresses:

$$\sigma_1 = \sigma_{\text{avg}} + R = -0.968 + 2.047 = +1.079 \text{ ksi} \quad \text{(Tension)}$$

$$\sigma_2 = \sigma_{\text{avg}} - R = -0.968 - 2.047 = -3.015 \text{ ksi} \quad \text{(Compression)}$$

Principal Angle (θ_p):

$$\tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(1.804)}{0 - (-1.937)} = +1.863$$

$$2\theta_p = \arctan(1.863) = 61.77^\circ \implies \theta_{p1} = +30.89^\circ \quad \text{(counter-clockwise)}$$

Step 7: Maximum In-Plane Shear Stress and Orientation

Maximum Shear Stress:

$$\tau_{\max} = R = 2.047 \text{ ksi}$$

acting with an average normal stress of $\sigma_{\text{avg}} = -0.968$ ksi.

Orientation of Maximum Shear Plane (θ_s):

$$\theta_{s1} = \theta_{p1} - 45^\circ = 30.89^\circ - 45^\circ = -14.11^\circ \quad (\text{or } +75.89^\circ)$$

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